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Trend of Presenting Concept of Fractions in the Mathematics Textbooks of the Second to Fourth Grades of Primary Schools in Iran and Japan

Narges Yaftian [1](Corresponding author)
Fatemeh Abbasi [2]

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ABSTRACT

The purpose of this research was to compare the trend of presenting concept of fractions in second to fourth grades mathematics textbooks in Iran and Japan, and also to examine what topics of fractions these textbooks present. The method was comparative-qualitative research, the statistical population was the countries of the world, and the selected sample was the two countries of Iran and Japan. The strategy of choosing countries was "different social systems, different educational outputs". The data collection method was done through the examination of primary and secondary sources, such as mathematics textbooks of Grade 2-4 of primary schools, related articles and books, and available documents in official and government databases. Bereday Model was used to analyze the data. The results showed that the concept of fraction has been introduced in two countries from the second grade and using the part-whole subconstruct, and these textbooks have similarities and differences in presenting the concept of fraction. Among these similarities, we can mention the concept of fractions greater than unity and mixed numbers. Among the non-common topics, we can mention the discrete representation of the part-whole subconstruct in Iranian textbooks and the conversion of a fraction larger than a unit to a mixed number and vice versa in Japanese textbooks. The lower performance of Iranian students in the concept of fractions in TIMSS compared to Japan shows that in the common topics, only a greater number of pages and more problems in the textbook will not help, and it is necessary to review and revise Iran's mathematics textbooks in common topics on the subject of fractions.

[1] Assistant Professor, Department of Mathematics, Faculty of Science, Shahid Rajaee Teacher Training University, Tehran, Iran, Email: yaftian@sru.ac.ir

[2] Master of Science in Mathematics Education, Department of Mathematics, Faculty of Science, Shahid Rajaee Teacher Training University, Tehran, Iran, Email : farhangian94fatemehabbasi@yahoo.com

1. Introduction

Fractions are one of the most complex mathematical concepts that primary students learn. It is also the core and challenging part of the primary mathematics curriculum (Fauzi & Suryadi, 2020; NCTM, 2000; Pedersen & Bjerre, 2021; Purnomo, Widowati & Ulfah, 2019; Wijaya, 2017). Studies on the concept of fractions have shown that a deep understanding of this concept provides the basis for advanced mathematics learning (Braithwaite, Pyke & Siegler, 2017; NCTM, 2000) and if students do not have a good understanding of it, their ability to learn other related mathematical concepts, such as measurement, algebra, statistics and probability, will be severely limited (Cramer, Post & delMas, 2002; Tsai & Lee, 2017; Yang, Reys & Wu, 2010). Fractions are a multifaceted concept that consists of five interrelated subconstructs: part of the whole, measure, operator, quotient and ratio (Kieran, 1976). Kieran was the first person who pointed out the multiple nature of this concept and its subconstructs. Figure 1 summarizes the mentioned subconstructs.

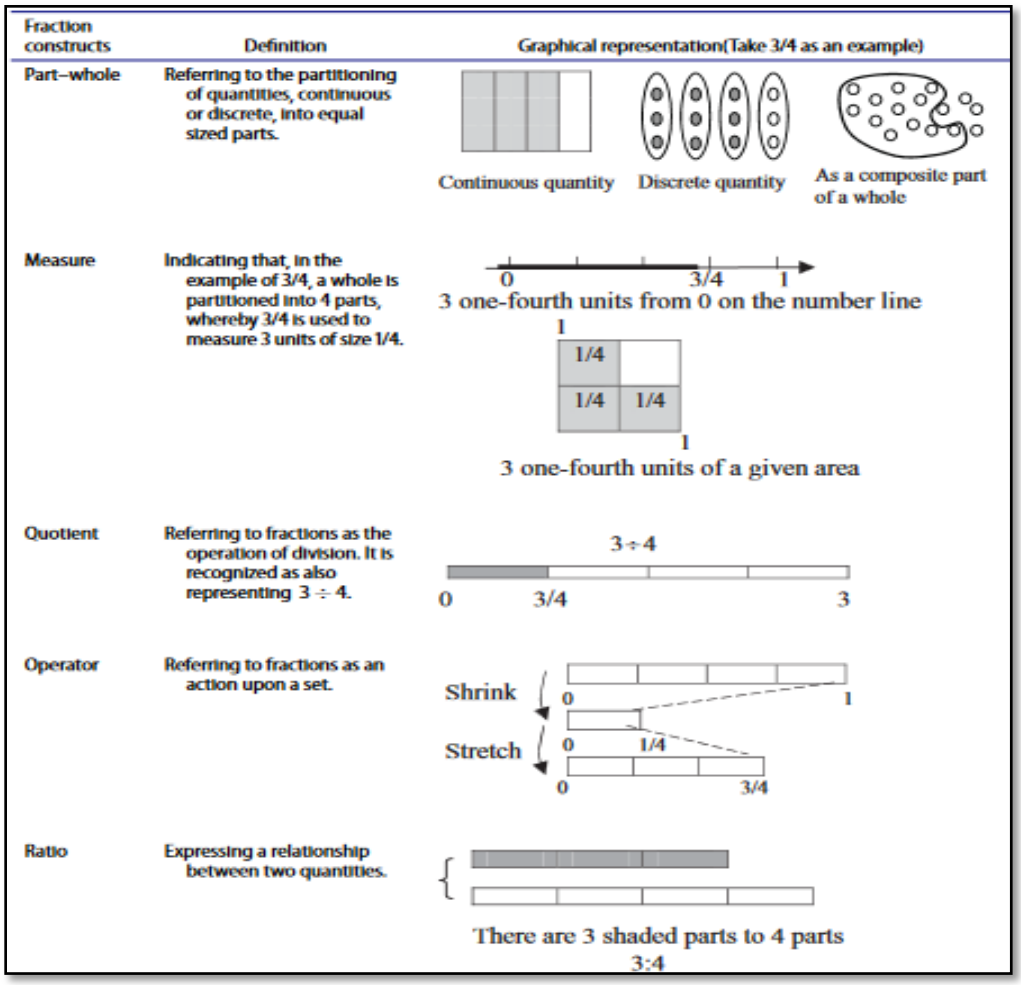

Figure 1: The five subconstructs of fractions (Tsai and Li, 2017, p. 247)

According to Figure 1, in the part-whole subconstruct, a continuous or discrete quantity can divide into equal sized parts. For example, ¾ can be three pieces eaten out of four equal pieces of a pizza (continuous model) or shows 3 black beads out of 4 beads (discrete model).

In the past decades, researchers have identified several factors as influencing factors on learning fractions, and the results of numerous internal and foreign studies show that students face many challenges in understanding the concept of fractions and solving problems related to them (Izadi & Reyhani, 2020, 2021; Barmby, Harries, Higgins & Suggate, 2009; Charalambous, Delaney, Hsu & Mesa, 2010; Doosti, 2013; Fauzi & Suryadi, 2020; Newton, Willard & Teufel, 2014; Purnomo et al., 2019). Fauzi and Suryadi (2020) in their research showed that some difficulties of students in the concept of fractions and solving its problems are: the inability to recognize and present a fraction for the given shape, the inability to understand the concept of fractions as a part-whole, and having misconceptions in the subconstruct of the operator. Newton et al. (2014) also pointed out in the study of students' challenges and misconceptions in the concept of fractions that most students have difficulty in explaining the concept of multiplying two fractions and solving related problems. For example, when multiplying two fractions, they first find a common denominator between the two given fractions and then calculate the product. The results of internal studies also indicate that some of the difficulties and misconceptions of Iranian students include 1) understanding the fraction as two separate numbers, a denominator is an arithmetic number and the numerator is a part of the denominator; 2) difficulty in distinguishing the numerator as a part of the whole and the denominator as a whole and understanding the fraction as a ratio of part to part; 3) difficulty in making connections between different representations of fractions; 4) difficulty in placing fractions on the number axis due to not having a correct understanding of fractions as numbers; 5) difficulty in solving unusual problems about fractions. 6) using approximate division to calculate and present a fraction for a given shape (Izadi & Reyhani, 2020, 2021; Doosti, 2013; Reyhani, Bakhshalizade & Doosti, 2014).

Since there is no native assessment framework in the instructions of the Iranian educational system to determine the level of performance of Iranian students in various mathematical concepts, including the concept of fractions, examining the level of performance of Iranian students in solving problems related to fractions in the international study of the Trends in International Mathematics and Science Study (TIMSS), as a valid international study, provides an acceptable and reliable view of the level of learning of Iranian students in this concept. From 1995 to 2019, Iran participated in all seven periods of the comparative study of TIMSS. TIMSS is conducted to evaluate the quality of the educational systems of the participating countries and to improve the level of learning in their

educational systems (Kiamanesh, 2022), and the participation of Iranian students in this study can provide a perspective of the efficiency and effectiveness of the Iranian educational system and textbooks. In different periods of conducting TIMSS, Iran has had a low performance compared to the international average. This described situation in Iran is considered a challenge in the primary education system (Kiamanesh, 2022; Sadeghi et al., 2021), which must be solved by necessary steps. Examining and tracking the results of Iranian students in solving problems related to fractions in published problems of TIMSS in different periods show that Iranian students have relatively low performance in solving these kinds of problems (Bakhshalizade& Kashefi, 2017; Mullis, Martin, Foy, Kelly & Fishbein, 2020). Figure 2 shows a fraction problem from the TIMSS 2015.

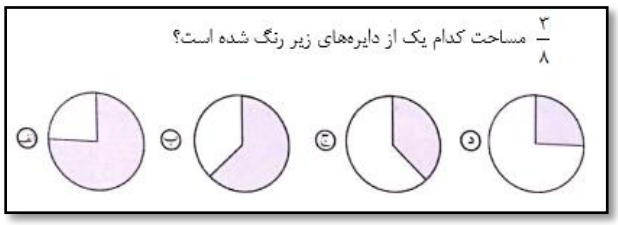

Figure 2: Question with code M05-04A from published problems TIMSS 2015 (Bakhshalizade & Kashefi, 2017)

In this problem, students are asked to select a shape in which the area of the painted part is three-eighths of the whole circle (Bakhshalizade& Kashefi, 2017). The presented results of this problem show that the international average for responding to this problem is 44%; meanwhile, the average percentage of correct responses for Iranian students is about 32%. Examining the results presented from other problems published on the topic of fractions in TIMSS 2015 and also TIMSS 2019 show that Iranian students do not have an acceptable situation in solving such problems (Bakhshalizade& Kashefi, 2017; Mullis et al., 2020). Therefore, it is necessary to investigate the causes and act accordingly to solve the difficulty.

Barmby et al. (2009) believe that in addition to the multiple nature of the concept of fractions, which can make it difficult for students to learn it, students' misconceptions about understanding fractions as numbers and how to operate with them is also relevant to the methods of presenting and teaching this concept. Other studies in this field have also confirmed this situation and revealed that students' difficulties in learning fractions are related to the complexity of this concept and the educational approaches that have been used for teaching fractions in the textbooks (Alajmi, 2012; Bahr, Harel, Post & Lesh, 1993; Charalambous et al., 2010; Fauzi & Suryadi, 2020). Therefore, it can be said that mathematics textbooks, as one of the main educational tools and widely used resources

for teaching and learning students (Van den Ham & Heinze, 2018), may play a role in causing trouble or eliminating difficulties. Various studies have been conducted on the relationship between textbooks and students' achievements and their performance at the international level, all of which have confirmed the impact of textbooks on student performance (Grouws, Tarr, Chávez, Sears, Soria, & Taylan, 2013; Tarr, Grouws, Chávez & Soria, 2013; Van den Ham & Heinze, 2018). Therefore, mathematics textbooks should be considered an important variable.

One of the ways used by researchers in the field of mathematics education to understand the differences in the education and success of different nations in international studies, including TIMSS, is to analyze the textbooks of different countries and compare them with each other (Charalambous et al., 2010). Therefore, due to the fundamental role of this vital educational resource, many researchers believe that focusing on the review and comparison of textbooks from different countries will reveal some of its possible inadequacies and deficiencies (Fan, Zhu & Miao, 2013; Hong & Choi, 2014) and finally the curriculum planners and authors plan to solve it. Therefore, examining and comparing the mathematics textbooks of different countries in the way of presenting and teaching the concept of fractions with the countries that have a high performance in international studies can be useful for benefiting from experiences and getting to know the strengths and weaknesses and finding ways to solve possible difficulties. While Iran has performed lower than the international average in different periods of TIMSS, Japan has always obtained an acceptable average score and rank among the participating countries (Bakhshalizade& Kashefi, 2017; Mullis et al., 2020). The unfavorable performance of Iranian students in facing problems related to fractions in the TIMSS during the years of conducting this study and on the other hand, the favorable performance of a country like Japan (Mullis et al., 2020) makes the structure and way of presenting the concept of fractions also in the mathematics textbooks of this country and its comparison with the textbooks of Iran should be important. For a brief introduction to the Japanese education system, it can be said that education is compulsory for 9 years and primary education includes grades 1 to 6 (from 6 to 12 years old) (Takeuchi & Shinno, 2020). Japanese textbooks are strictly curriculum-based, as mandated by the Japanese Ministry of Education ((Takeuchi & Shinno, 2020). In Japanese primary schools, six different series of mathematics textbooks are used for primary school, each of which is published by different companies and publishers. Local commissions of education are responsible for selecting school textbooks, and these commissions announce their selection on their official website or in other documents (Isoda & Olfos, 2021). Regarding the educational system in Iran, it can be said that, like Japan, primary education includes grades 1 to 6 (from the age of 6 to 13) (Sadeghi et al., 2021) and textbooks are written according to

the order of the Ministry of Education and according to the curriculum. Unlike Japan, in Iran, a collection of textbooks is used throughout the country (Sadeghi et al., 2021).

Since different learning opportunities lead to different learning outcomes and textbooks play a significant role in creating these opportunities (Hadar, 2017; Van den Ham & Heinze, 2018), preparing and composing a high-quality mathematics textbook is a critical issue for authors of textbooks in different countries. Therefore, in recent years, comparative studies of textbooks have become very popular among mathematics education specialists (Bütüner, 2021). The comparative studies conducted on the textbooks of different countries are multifaceted and include various dimensions. Many of these studies have investigated mathematical processes such as problem solving, problem posing, and reasoning (Cai & Jiang, 2017; Fan & Zhu, 2007; Fujita & Jones, 2014; Karami Zarandi, 2009; Miyakawa, 2017). For example, Karami Zarandi (2009) in her research examined the teaching of the problem solving process in the secondary school curriculum of the United States, Australia, Japan, Singapore and Iran. The results indicate that problem solving is an integral part of the curriculum of four countries, the United States, Australia, Singapore, and Japan, and it is not viewed as a separate unit of materials in these four countries, but is taken as a method for teaching mathematical content. On the other hand, problem solving in Iran has been isolated and not enough attention has been paid to it.

Also, some comparative studies have focused on curricula, a specific mathematical content or specific mathematical problems in the textbooks of two or more countries (Alajmi, 2012; Azadi & Shiravani Shiri, 2020; Bütüner, 2020, 2021; Danaei Zarchi, 2018; Glasnovic Gracin, 2018; Izadi, Ahadi & Reyhani, 2015; Jones & Fujita, 2013; Kian, Danaei Zarchi & Zandvanian Naeini, 2019; Moradi, Khoshbakht & Alborzi, 2016; Mehrjoo, Nourian, Norouzi & Abai Kopai, 2022; Szitányi, Biró & Csíkos, 2020; Takeuchi & Shinno, 2020). For instance, in their research, Takeuchi and Shinno (2020) compared the contents of Japanese and England mathematics textbooks, focusing on the concepts of symmetry and transformations at the lower secondary level. The results of their research showed that symmetry and transfer (transformations) in Japanese textbooks were strongly influenced by the teaching of geometric proofs, while teaching the concept of transformation in England textbooks had many connections to other contexts or contents across domains. Some other studies have been conducted by focusing on examining the physical and appearance structures of textbooks and their text (Alajmi, 2012; Ginsburg, Leinwand, Anstrom & Pollock; 2005). For instance, Ginsburg et al. (2005) in their research, in addition to examining the key features of the primary school mathematics education systems in Singapore and the United States, also reviewed the structures of mathematics textbooks at this level and issues such as how

textbooks are organized in different chapters, lessons and also, they checked the number of pages. Other studies have investigated the general or cultural characteristics of textbooks (Clivaz & Miyakawa, 2020; Haggarty & Pepin, 2002). For instance, in their research, Klivaz and Miyakawa (2020) investigated the cultural characteristics of a mathematics lesson using concrete examples of lesson design and implementation and how mathematics lessons are formed and produced inside and outside the classroom or school in a specific country.

Among the comparative studies that have been conducted on specific mathematical content in the mathematics textbooks of different countries, numerous internal and foreign studies have examined and compared the concept of fractions in the textbooks of different countries (Alajmi, 2012; Charalambous et al., 2010; Hwang, Yeo & Son, 2021; Tajari, 2012; Yang, 2018; Vula, Kingji-Kastrati & Podvorica, 2015). Also, due to the impressive performance of Japan in international assessments including TIMSS, internal and foreign studies have also compared Japanese primary school mathematics textbooks with other countries in the concept of fractions (Alajmi, 2012; Danaei Zarchi, 2018; Hwang et al., 2021; Kian et al., 2019; Tajari, 2012). Alajmi (2012) in his study focused on the physical characteristics of the textbooks, the structure of the lessons offered, the trend of presenting the subject of addition and subtraction, the part-whole subconstruct of fractions and the nature of the mathematical problems presented in Kuwait, the United States, and Japan. He showed that the nature of the presented problems and the structure of lessons on this subject in these three countries have similarities and differences with each other. For example, fractions in the US and Kuwait mathematics textbooks introduce in the first grade of primary school, while Japanese mathematics textbooks are not introduced in the first grade. Also in Japanese textbooks, they use linear models to discuss fractions, and relate fractions to measurements. Textbooks from the United States use concrete teaching aids to help students learn fractions, and selected textbooks from Kuwait use continuous representations to illustrate fractional ideas.

One of the comparative studies that has been done on the subject of studying on the division of fractions and their significance in primary school can be referred to the study of Tajari (2012). In this research, three topics of organizing, conceptualizing and presenting content and practicing problems for teaching and learning how to divide fractions, have been addressed in the textbooks of four countries; Iran, the USA, China and Japan. The results of this study indicate that in the exercises section, the textbooks in China and Japan have the most, and the textbooks in Iran have the least diversity, and in the United States, in addition to the vast variety, many of the activities in the textbooks allocated to the repetitive exercises. For teaching the concept of dividing fractions, the Japanese and Chinese textbooks use pictorial representations, linguistic and numerical

descriptions. Textbooks in the United States also use word problems, linguistic explanations, and numerical expressions to complete the process of calculating the division of fractions. In another internal research, Kian et al. (2019) examined the content of the fourth grade mathematics curriculum in Iran and the five pioneering countries of TIMSS 2015, namely Japan, Singapore, South Korea, Hong Kong, and Taiwan. The results of their research indicated that some concepts in the mathematics curriculum of the fourth grade of these countries, such as the concepts of numbers, calculation, measurement and geometry, are similar. In addition, the results showed that the studied curricula differ from each other in terms of mathematical problem solving skill, analysis and use of mathematical subjects, statistics and algebra, as well as a positive attitude towards learning mathematics.

As it was mentioned earlier, in the research conducted on the topic of fractions in Iran, some studies have investigated students' perceptions of the concept of fractions, students' misconceptions about this concept and students' performance in different fractional subconstructs (Izadi & Reyhani, 2021; Doosti, 2013; Reyhani et al., 2014). However, the review of the research literature indicates that comparative research has rarely been done on the subject of examining the process of presenting the concept of fractions, and a comprehensive internal research has not yet investigated and compared the presentation of the concept of fractions in the primary school mathematics textbooks of Iran and other successful countries in international studies. Considering the importance of the concept of fractions, conducting research focused on comparing the process of presenting the concept of fractions in Iran's mathematics textbooks with other pioneering countries in international studies, including Japan, is a necessity considering the successive successes of Japanese students in international studies such as TIMSS. Therefore, due to the difficulties of most Iranian students in learning and understanding the concept of fractions, the present research with the aim of investigating and comparing the process of presenting the concept of fractions in second to fourth grades mathematics textbooks in Iran and Japan, because of conducting TIMSS in the fourth grade, it has been done. In the present research, the first grade mathematics textbook from both countries was not examined because the concept of fractions for the first time in the second grade of Iran and Japan is presented. In this research, the following questions are responded:

- What is the trend of presenting the concept of fractions in the mathematics textbooks of the second to fourth grades of primary school in Iran, and what concepts of fractions do students become familiar with until the fourth grade?

- What is the trend of presenting the concept of fractions in the mathematics textbooks of the second to fourth grades of primary school in Japan, and what concepts of fractions do students become familiar with until the fourth grade?

- What are the similarities and differences in presenting the concept of fractions in the second to fourth grades mathematics textbooks of Iran and Japan?

2. Research Method

The purpose of this study was to investigate and compare the similarities and differences in presenting the concept of fractions in the second to fourth grades mathematics textbooks of Iran and Japan. Also, it examines that students of both countries are familiar with which concepts and topics of fractions until the fourth grade. The current research is among the qualitative studies in terms of its purpose and comparative in terms of its method. The statistical population includes all the countries of the world and the selected sample, including the two countries of Iran and Japan, and the strategy of selecting countries is "different social systems and different educational outputs". Also, the content under examination is the content of second to fourth grades mathematics textbooks in Iran and Japan in the chapters related to fractions. Japan has been purposefully selected for comparison with Iran based on three main criteria. Both Iran and Japan have a national curriculum, and Japan has a national curriculum called Course of Study, which is usually revised every ten years. In addition, these two countries are located in Asian continent. Another reason for choosing Japan to compare with Iran is the level of Japanese students' performance in solving problems related to fractions in the international study of TIMSS. In the present study, for Japanese primary school mathematics textbooks, the Tokyo Shoseki series of 2012 were used, which are the most widely used in Japanese primary schools (Alajmi, 2012; Isoda & Olfos, 2021), and their translation in English is also available. Since the curriculum in Japan is revised every ten years and the latest changes in the textbooks based on the curriculum occurred in 2012, then the Japanese textbooks published in 2012 were the basis of this study. For Iran, primary school mathematics textbooks published in 2022 have been used.

The method of data collection is documentary that was done through the examination of preliminary sources such as mathematics textbooks of the second to fourth grades of two countries, and secondary sources such as related articles and books and documents available in official and government databases. To analyze the data and present the results, George Bereday's four-stage model including the four stages of description, interpretation, adjoining and comparison, and agreement and disagreement of John Stuart Mill has been used. The data collection tools are checklists whose formal and content validity has been approved by three professors of

mathematics education at Shahid Rajaee Teacher Training University and Farhangian University of Tehran, as well as three experienced primary school mathematics teachers in Tehran province.

The steps of conducting this research were as following. In the first step, describing, the structure of mathematics textbooks of both countries in presenting concepts, the number of pages in each textbook, the number of chapters in each textbook, the number of chapters related to fractions in each textbook, and the percentage of pages dedicated to this concept were determined in the second to fourth grades. Then, the process of presenting this concept and its content in the primary level textbooks of these two countries was categorized based on the grade and in tables. In the interpretation step, the described and collected information that was systematically noted from the previous step, along with the performance of Iranian and Japanese students in solving problems related to fractions in the international study of TIMSS, were interpreted and analyzed to be able to judge correctly about the similarities and differences in presenting of the concept of fractions in the primary school mathematics textbooks of the two countries and the effect of presenting these materials on the performance of students in the international study of TIMSS. In the adjoining step, the classification and categorization of different concepts of fractions in the reviewed textbooks were done separately for the two countries of Iran and Japan. In the following, the obtained data were classified close to each other for the two countries. In this step, it was tried to classify the obtained information and in some cases to arrange it in the form of a table, which can be used for comparison in the next step. Finally, in the last step, i.e. comparison, the information categorized based on the research question, was analyzed and the differences and similarities in the presentation of the concept of fractions in the second to fourth grades mathematics textbooks of the two countries were presented.

3. Findings

A) Description and Interpretation

1. Iran

The structure of second to fourth mathematics textbooks is almost similar to each other. Each chapter begins with a title page containing an image and text (theme) that prepares students to learn the topic of each chapter. The presence of this page helps teachers to give students the necessary motivation to start lessons. The content of each chapter of the second to fourth grades mathematics textbooks in Iran consists of three sections: activity, classroom practice and exercise, and each is included in the textbook with specific purposes, which is presented in the following way: activity-classroom practice-exercise. What is intended for each activity is to familiarize the

students with the concept of the lesson and to participate them in understanding the required knowledge. Classroom practices are to stabilize, deepen, and in some cases, generalize and cocompletetudents' learning. Those practices get reviewed in the classroom. Exercises given to students as their homework, so doing them would be their responsibility**.**

In Table 1, the number of textbook pages for each grade of Iran, the number of chapters in each textbook, the number of chapters related to fractions, the percentage of pages dedicated to chapters related to fractions, and the number of lessons dedicated to fractions are examined.

Table 1: Number of chapters and pages containing fractions in Iranian textbooks based on grades

Grade	Number of page	Number of chapters	Number of fraction chapter	Percent of page with fraction	Lessons devoted to a fraction
2	144	8	7	12.5	2
3	150	8	3	12	4
4	154	7	2	14.28	4

Table 1 shows that Iranian primary school students learn about fractions in the last chapters of second grade textbooks. But in the third and fourth grades, familiarity with fractions is in the beginning chapters of the textbook. The number of lessons dedicated to teaching fractions and also the pages related to teaching this concept in the textbook increase from the second to the fourth grades. The introduction of fractions in accordance with the presented structure begins with the part-whole subconstruct of fractions. Iran has introduced this concept in the second grade, chapter seventh. Figure 3 shows the first activity to start teaching the concept of fractions from the second grade mathematics textbook.

Figure 3: Activity on page 110 of Iran's second grade mathematics book

As we can see from Figure 3, in the opening activity of this chapter, students are measuring the length of an ant, which is slightly less than a centimeter. They should decide how to express this part of a centimeter. The lesson introduces the concept of fractions to provide a solution for measuring the length of an ant. This solution uses a measurement model that represents a fraction of a centimeter. After practicing in this topic, students will learn how to divide shapes into several equal parts. In the third grade, while being familiar with the symbol of fractions, students through different activities and exercises face fractions as the measure in different representations (relation of fractions with length, hour and weight). In the fourth grade, the mixed number and its symbol are introduced to the students. Multiplication of integers in fractions and comparison of fractions with unequal numerators and denominators is also one of the goals of this textbook. One of the interesting things that the fourth grade textbook deals with is the presentation of a discrete representation of fractions**.** Table 2 examines the trend of presenting the concept of fractions and operations on it through the second to fourth grades Iran mathematics textbooks.

Table 2: Goals and educational content of fractions in the Grades 2-4 of primary schools of Iran

Textbooks	Title of Chapter	Goals and content
Second grade	Possibility and fraction	Familiarity with a part of a unit, familiarity with equal division of objects and shapes, familiarity with the concept of fractions and its verbal expression, familiarity with the concept of probability as an application of fractions, familiarity with less and more probability, relationship between probability and fractions, expressing the probability as a fraction
Third grade	Fractional numbers	Familiarity with the fractions symbol, Familiarity with the fractional line and numerator and denominator in fractions such as vinculum, numerator and denominator, writing the right fraction for shapes, how to use fractions in measurement units such as the connection between fractions and length, clocks and weight units, introducing equal fractions and representing them on shapes and axis, comparing fractions with equal numerators and denominators, introducing half, quarter and their equivalent fractions
Fourth grade	Fraction	Defining the mixed number and introducing its symbol and how to write it, expressing the fractions for discrete shapes for the first time, understanding the need to introduce a unit for discrete shapes, introducing zero fraction and its relation to unit, introducing mixed number symbol with different representations, adding and subtracting two fractions with common denominators, understanding unit and zero fraction by adding and subtracting fractions, writing equal fractions with fraction in discrete and continuous shapes, writing fractions equal to one fraction and find the fourth component

		of an equality, simplifying fractions, using equality of fractions to equal denominators in comparison and addition and subtraction, introducing one third and one fifth and their equivalent fractions

According to Table 2, we understand the trend of presenting the concept of fractions in each grade, and finally, by the end of fourth grade, which concepts of fractions and operations on them, students get familiar with. Also, it can be said that the concept of fractions in Iran is introduced for the first time in the second grade of primary school, and students in this grade are only familiar with the verbal representation of fractions. In the second grade, there is a general introduction to the concept of fractions, how to divide continuous forms into equal parts, and the relationship between fractions and the subject of probability, but in the third grade, the fractions symbol is introduced. The operator subconstruct has been introduced to students from the third grade. In the second grade, the subconstruct of measure is first conceptualized for students, and in the third grade, students are introduced to measure the subconstruct of fractions in different representations. In the fourth grade, the introduction of the mixed number and its symbol, as well as the addition and subtraction of fractions with equal denominators are included in the mathematics textbook.

In the following, after familiarizing with the content presented in the chapters related to fractions in second to fourth grades mathematics textbooks, for a better understanding of the performance of Iranian students in this concept, two examples of the problems published by TIMSS 2015 and 2019 along with their results for the country of Iran are mentioned. The data presented from the last TIMSS held, TIMSS 2019, shows that Iran has been ranked 50th out of 58 countries and six participating states (Mullis et al., 2020). In the other periods of TIMSS, Iran also has always had a relatively low rank and performance compared to the international average. Figure 4 shows the average performance of Iranian students over the years of TIMSS.

Figure 4: The trend in the mathematical performance of Iran's fourth grade students (Mullis et al., 2020)

After the changes in the educational system and textbooks in Iran and after participating in TIMSS 2011 and 2015 study, there has been no positive change in the score and rank of Iranian students, and it seems that the changes in textbooks have not played a role in the score and rank of students. Examining the published problems from TIMSS 2015 as well as TIMSS 2019 on the subject of fractions also show that Iranian students are weaker in solving most problems than the international average. For example, in a problem related to the comparison of fractions with equal numerators from TIMSS 2015, the difference between the performance of Iranian students and the international level is considerable (Bakhshalizade& Kashefi, 2017). The problem refers to the concept of fractions, which was among the topics raised in the third grade mathematics textbook of Iran (Table 2), but the information provided by TIMSS site shows that only 30.3% of Iranian students were able to respond to this question correctly; while this percentage is 50.1% at the international level. This difference in the average percentage of correct responses is notable. It shows that despite addressing this concept and providing examples in the textbooks, Iranian students are facing difficulties in this concept compared to the international average.

Because only ten mathematical problems of TIMSS 2019 have been fully published, it has not been possible to fully examine the performance of Iran in problems related to the concept of fractions in this study. One of the published problems of TIMSS 2019 on the concept of fractions is a problem with the calculation of fractions remaining from the whole path (Mullis et al., 2020). In this problem, the appropriate mathematical operation to reach the correct response is subtracting a

non-unit fraction from 1 (unite fraction). By examining the process of presenting the concept of fractions and operations on it for the country of Iran, it can be seen that this topic is one of the topics that Iranian students have been familiar with until the fourth grade and they have solved problems from this topic in their textbooks. The published results for this problem show that the international average for correct responses is 47% and the average percentage of correct responses for the country of Iran is 41%.

2. Japan

The trend of presenting lessons and concepts in each chapter of the second to fourth grades mathematics textbooks in Japan is as follows: each chapter begins with an introductory problem that identifies the importance and application of that concept. In the following, the purpose of the lesson is introduced to the students, and then some tips are offered to them for solving the problem. The textbook afterward usually provides a summary of the lesson in a box with the same title. Finally, at the end of each lesson and chapter, there are more exercises for students to practice. Table 3 shows the number of pages in Japanese mathematics textbooks for each grade, the number of chapters in each textbook, the number of chapters on fractions, the percentage of pages devoted to chapters on fractions, and the number of lessons devoted to fractions.

Table 3: Number of chapters and pages containing fractions in Japanese textbooks based on grades

Grade	Number of pages	Number of chapters	Number of fraction chapter	Percent of pages with fraction	Lessons devoted to fraction
2	197	17	11	1.5	2
3	215	19	14	5.58	4
4	239	16	12	5.02	4

According to Table 3, it can be seen that Japanese students get familiar with fractions from the second grade and in the final chapters of their textbooks. The content related to the concept of fractions also increases from the second to the fourth grades. What is interesting at the beginning of teaching the concept of fractions from the second grade Japanese mathematics textbook is the connection between fractions and real-world problems centered on dividing objects into equal parts. Figure 5 shows the initial problem for entering the subject of fractions in the second grade Japanese mathematics textbook.

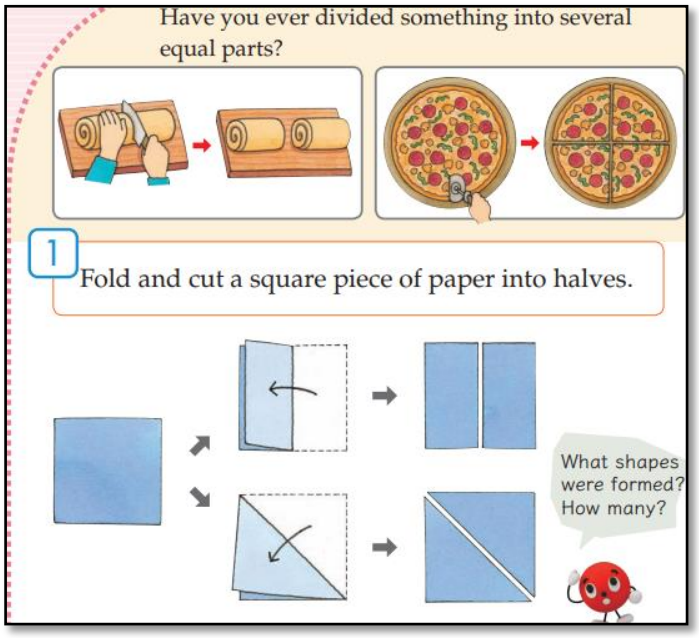

Figure 5: The beginning activity of starting to teach fractions, page A106 of the second grade Japanese mathematics textbook

The second grade mathematics textbook begins teaching fractions by asking a question. The students are asked to talk about their experiences of dividing objects into several equal parts, and then use this topic to identify a part of a unit as a fraction. Next, in the form of different activities, it practices with students how to divide shapes equally. Later, in the third grade, while getting acquainted with continuous representations of fractions, it introduces them to the relationship between fractions with 10 as denominators and decimal numbers, and soon students learn how to add and subtract fractions with equal denominators. The fourth grade mathematics textbook covers more content on the concept of fractions and operations on them than the second and third grades textbooks for students. A review of Japanese mathematics textbooks in the second to fourth grades showed that most of the initial problems presented to introduce a concept related to fractions are focused on measuring length, and most of the problems raised in the third and fourth grades are about measuring length (distance) and volume of liquids and the presented models are measuring in meter and liter units. Such models establish a proper connection with students' lives and make the subject of fractions meaningful for students. Japanese mathematics textbooks also use the axis of numbers in the topic of fractions. The number axis helps students visualize fractions and understand the differences in size and order of fractions. Table 4 is placed to learn more about the

process of presenting the concept of fractions and operations on them in textbooks of the second to fourth grades in Japan.

Table 4: Goals and educational content on the subject of fractions in the second to fourth grades of Japan

Textbooks	Title of Chapter	Goals and educational content of fraction chapter
Second grade	Fraction	conceptualization of fractions as a part-whole considering the number of desirable parts of a whole that is divided into equal parts, familiarity with equal division of shapes, familiarity with numbers such as 1/2 and 1/4 as a fraction
Third grade	Fraction	The meaning of fractions and how they are represented, continuous representations of fractions, fractions as measure, relations of fractions with denominators of 10 and decimal numbers, addition and subtraction of fractions with equal denominators
Fourth grade	Fraction	introducing fractions unit and smaller and greater than unit, introducing mixed number, converting fractions greater than unit to a mixed number and vice versa, introducing equal fractions, comparing fractions with equal numerators, adding and subtracting fractions greater than unit and mixed number

In Table 4, you can see the goals and educational content of the chapter related to fractions in the textbooks reviewed from Japan. According to Table 4, we can see to what extent students in each grade become familiar with the concept of fractions.

The presented process of teaching fractions in primary school mathematics textbooks in Japan shows that the students of this country are familiar with equal division of continuous shapes and considering a part of a whole as a fraction from the second grade. Introduction of fractions, fraction symbol, numerator and denominator in fractions, familiarity with fractions as measure and the relation of fractions with denominators of ten and decimal numbers are reserved for the third grade mathematics textbook. Also, familiarization with addition and subtraction of fractions with equal denominators is provided in the third grade. In the fourth grade, while familiarizing with fractions smaller than one, larger than one, and unit fractions, mixed numbers and their definition are discussed. Also, converting a fraction larger than unit to a mixed number, a mixed number to a fraction larger than unit, addition and subtraction of fractions larger than unit and mixed numbers are among the abilities that the students of this grade achieve.

After familiarizing with the content presented in the chapters related to fractions in second to fourth grades mathematics textbooks in Japan, to better understand the Japanese students' performance, two examples of problems published by TIMSS 2015 and 2019 on the concept of

fractions, along with their results for Japan are mentioned. Examining the trend of the average performance of Japan during the years of conducting TIMSS indicates that this Asian country has always had an acceptable rank and score compared to the international average as well as most of the participating countries. The results of the latest study of TIMSS, TIMSS 2019, also indicate that Japan ranked fifth among fifty-eight countries and six participating states (Mullis et al., 2020). Figure 6 shows the trend of the average performance of Japan in the years of TIMSS.

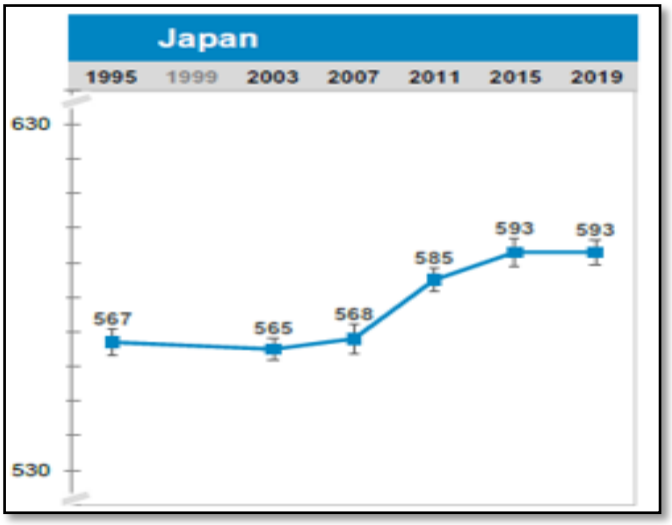

Figure 6: The trend in the mathematical performance of Japan's fourth grade students (Mullis et al., 2020)

In addition to the average and acceptable rank that Japan has as a whole, the examination of the results of Japanese fourth grade students in solving problems related to the concept of fractions also shows that these students have had an acceptable average score and rank among countries in solving such problems. For example, in the problem of comparing fractions with equal numerators from TIMSS 2015, the average of correct responses for Japan was 71% and at the international level, it was 50.1%. Of course, it should be noted that Japanese students were introduced to the content presented in this problem in the fourth grade (Table 4). In another problem with the subject of calculating the fraction remaining from the path from TIMSS 2019 study, which has familiar content for Japanese students, Japan has an average performance of 76, while the international average for solving this question was 47%.

B) Juxtaposition and Comparison

After describing and interpreting the process of presenting the concept of fractions in second to fourth grade mathematics textbooks of Iran and Japan, the similarities and differences between the two selected countries in the concept of fractions have been discussed by Juxtaposition and Comparison. First, it is examined that in each textbook from Iran and Japan, the topic of fractions occupies how many chapters and what percentage of the pages of each textbook and it is determined that the emphasis of the textbook on this concept was more or less in which textbook and from which country. Table 5 shows the number of textbook pages for each grade, the number of chapters in each textbook, the number of chapters related to fractions, the percentage of pages devoted to chapters related to fractions, and the number of lessons devoted to fractions.

Table 5: Number of chapters and pages containing fractions in Iranian and Japanese textbooks based on grades

Country	Grade	Number of pages	Number of chapters in each book	Number of fraction chapter	Percent of pages with fraction	Lessons devoted to fraction
Iran	2	144	8	7	12.5	2
	3	150	8	3	12	4
	4	154	7	2	14.28	4
Japan	2	197	17	11	1.5	2
	3	215	19	14	5.58	4
	4	239	16	12	5.02	4

As you can see in Table 5, in both countries textbooks, there is only one chapter applied to the concept of fractions. In the second grade mathematics textbook of both countries, the chapter related to fractions presents in the final sections of the textbook. In the third and fourth grades mathematics textbooks of Iran, the chapter on fractions presents in the initial part of the textbook, while in Japan, it is in the final part of their third and fourth grades textbooks. In both countries, the number of pages devoted to the fraction chapter adds up from the second to the fourth grades. In general, based on an overall outlook in Table 5, Japanese textbooks have more pages than Iranian textbooks, however the percentage of pages devoted to the topic of fractions in Japanese textbooks is much less than the Iranian textbooks. In the Japanese mathematics textbooks, the percentage of pages devoted to fractions in the second to fourth grades has increased from 1.5% to 5.02%. As well in Iran, it has growth from 12.5% to 14.28%. According to Table 5, the chapter related to fractions

in the second grade of Iran and Japan both has two lessons. Although, the percentage devoted to the topic of fractions in the second grade textbook of Iran is 12.5%, for the second grade of Japan is 1.5%. This discrepancy is because Iranian textbooks, unlike the Japanese textbook in the second grade, have more exercises to understand the fraction as a part-whole, to get familiar with the equal division of shapes and have a verbal representation of fractions. Examining the activities presented on the subject of fractions in the mathematics textbooks of the two countries show that most of the problems presented from this concept in the Japanese mathematics textbooks are closer to the students' real life than the problems presented in the Iranian mathematics textbooks. For students' deep and conceptual understanding of fractions, Japanese mathematics textbooks try to connect fractions with students' previous experiences and real life.

In the following, we compared the fractions related concepts in selected textbooks of Iran and Japan. First, common concepts on the subject of fractions are introduced in the textbooks of the two countries, and then the differences in the presentation of this concept in the second to fourth grades mathematics textbooks are clarified. After discussing the trend of presenting the concept of fractions in the second to fourth grades textbooks of both countries, we realized that parts of the curricula in both countries that include the concept of fractions, are actually similar. Those common concepts are specified in the following:

Conceptualizing the fraction as a part of the whole and presenting the fraction verbally by considering the number of desired parts of a whole that is divided into equal parts. Getting familiar with continuous representations of fractions, its symbol, fractional line and numerator and denominator in fractions, using problems focused on measuring the length and volume of liquids and the number axis to represent fractions, comparing fractions with the same numerator, having an understanding on fraction greater than unit and mixed numbers, adding and subtracting of fractions with equal denominators, introducing the mixed numbers and the way of presenting them, writing an equal fraction with another fraction.

After presenting the common contents of the concept of fractions in the reviewed books, the differences in the presentation of this concept are expressed. With an overall view to the content and educational goals of fractions in the second to fourth grades of Iran and Japan, the differences between the two countries in presenting some content concerned to fractions are tangible. Table 6 summarized the mentioned subjects.

Table 6: Differences in the presentation of the concept of fractions in the second to fourth grades mathematics textbooks of Iran and Japan

Concept	**Differences**	
	Iran	Japan
The connection between fractions and possibility	✓	×
Displaying approximate fractions for shapes	✓	×
The connection between fractions, weight and clocks	✓	×
Discrete representation of the fractions	✓	×
Simplifying fractions	✓	×
Adding and subtracting fractions greater than unit and mixed number	×	✓
Converting a fraction greater than unit to a mixed number and vice versa	×	✓
Addition and subtraction of fractions when the denominator of one fraction is a multiple of another one	✓	×
Multiplying the integer by the fractions	✓	×
Comparing fractions with the same denominators	✓	×

By looking at Table 6, we can see that there are subjects concerned to fractions that are presented in the second to fourth grades mathematics textbooks of Iran, and not in the Japanese textbooks or some concepts of fractions may have been presented for Japanese students in second to fourth grades, while Iranian students have not. For example, according to Table 6, we can see that Iranian students get familiar with the concept of fractions and probability, while Japanese students have not dealt with this subject. There is also this matter that Japanese students learn the method of converting a fraction greater than unit to a mixed number, and vice versa during the second to fourth grades textbooks, but Iranian students up to the fourth grade have not learned this skill yet. Another difference to mention in presenting the concept of fractions is the common concepts that both countries deal with through the second to fourth grades, but their presentation has taken place in different grades. Table 7 shows these differences.

Table 7: Differences in presenting different concepts of fractions in each grade

	The grade it gets presented	
Topics get presented in both countries	Iran	Japan
Writing equal fractions	Third grade	Fourth grade
Adding and subtracting fractions with the same denominators	Fourth grade	Third grade
Comparing fractions with same numerator	Third grade	Fourth grade
Understanding the part-whole in continuous representing of fraction	Second grade	In the second grade, a part of a whole that is divided into two, four and eight equal parts is introduced, and in the third grade, this issue gets expanded

According to Table 7, after comparing the textbooks of both countries, there are particular concepts of fractions that students in both countries learn up to the fourth grade, but this concept may not be taught in the same grade in both countries. For example, the subject of comparing fractions with the same numerator is one of the educational purposes of textbooks until fourth grade in both countries, although this topic brings up in third grade in Iran and in fourth grade in Japan.

In the following, the performance of Iranian and Japanese students in two problems with the content of fractions from TIMSS 2015 and 2019 are compared, which was presented as an example in the description and interpretation section. In the first problem of TIMSS 2015 with the common content of the third grade mathematics textbooks of Iran and the fourth grade of Japan on the topic of comparing fractions with equal numerators, it can be seen that only 30.3% of Iranian students were able to respond to this problem correctly, while this percentage at the international level is 50.1% and 71% for Japan. Also, the results of the problem from TIMSS 2019 on the topic of calculating the fraction remaining from the path, which has a common content of the concept of fractions for the two countries, show that the international average for responding is 47% and the average percentage of correct responses for the countries of Iran and Japan is equal to 41 and 76 percent, respectively. Although the performance of Iranian students to respond to this problem is close to the international level, the difference between their performance and Japanese students is still remarkable. Iran has performed lower than the international average and Japan in solving

problems with a common content of the concept of fractions for the two countries. Of course, it should be noted that these two problems were only samples of the published problems of TIMSS 2015 and 2019 on the subject of fractions.

4. Conclusion

Considering the importance and application of fractions in daily life and since it is regarded as a prerequisite for presenting and understanding many school mathematical concepts, it is necessary to teach it properly in primary school. In countries with a centralized education system, textbooks are one of the most important resources available for teaching and presenting different concepts to students and have a significant impact on their mathematical performance (Grouws et al., 2013; Tarr et al., 2013; Van den Ham & Heinze, 2018). Many researchers have approved that comparing the textbooks of different countries and the concepts presented in them can help planners identify the strengths and weaknesses of their country's mathematics textbooks (Fan et al., 2013; Hong & Choi, 2014). Based on this, the purpose of the present research is to investigate the process of presenting the concept of fractions in the second to fourth grades of Iran and Japan, and also pay attention to what topics of fractions the students of both countries are familiar with until the fourth grade.

During examining the trend of presenting the concept of fractions in the second to fourth grades mathematics textbooks of Iran, it was found that this country introduces fractions for the first time in the second grade of primary school, and to start teaching this concept, the textbook used the part-whole subconstruct, and this subconstruct is dominant in the process of teaching this concept in the textbook. In Iran, the content of each chapter of fractions in second to fourth grades mathematics textbooks consists of three parts: activity, classroom practice, and exercise, and presented in the same order. The activities presented at the beginning of teaching fractions are usually unrealistic (measuring the length of an ant to begin teaching the concept of fractions) and do not have much connection with the students' real life. From the second to the fourth grades, the content presented on the subject of fractions is increased in terms of volume and level of complexity of concepts, and students' knowledge of this concept and its subconstructs is completed.

Like Iran, Japan also teaches this concept from the second grade and uses the part-whole subconstruct. This result of the research is in line with the result of the study of Wijaya (2017), which showed that the part-whole subconstruct is the simplest and most common subconstruct of fractions that can be used when introducing this concept for the first time. Also, Doosti (2013) points out that the dominant approach in Iranian mathematics textbooks is based on the part-whole

subconstruct. In the trend of presenting the concept of fractions in Japanese textbooks, it can be said that the beginning of the teaching of this subject is by presenting a problem from real life, which determines the importance of presenting and teaching the concept of fractions. In the following, after guiding to solve the problem, a summary of the lesson is presented and more problems are introduced for practice and consolidation. The amount of content presented on the concept of fractions and their level of complexity increases from the second to the fourth grades, and in each grade, students are introduced to different topics of fractions.

A review of second to fourth grades textbooks of both countries showed that some concepts related to fractions are within the educational goals of both countries. Continuous representations of fractions, symbol of fraction (including fractional line, numerator and denominator), how to use problems focus on measuring the length and volume of liquids and the axis numbers for presenting fractions are some of those subjects that both countries students learn. Comparing fractions with equal numerators, learning about fractions greater than unit and mixed numbers, adding and subtracting fractions with equal denominators, teaching mixed numbers and the way of displaying them, and writing equal fractions are some of the other common topics in this field. This result of the research is consistent with some of the results presented in the research of Kian et al. (2019). Similarly, with this research, they showed that the students of Iran and Japan in the fourth grade become familiar with addition and subtraction of fractions with equal denominators, and these concepts are included in the set of goals of the fourth grade mathematics textbooks of both countries.

Also, the investigation and comparison showed that there are differences in the goals and the process of presenting the concept of fractions in the content of the textbooks of both countries. In presenting the concept of fractions and starting to teach this concept, Japanese mathematics textbooks use problems related to students' real and everyday lives. In contrast, in Iranian mathematics textbooks, the connection between the concepts and problems they presented with students' real life is less. and it is in line with the results of the research of Jafari Kafi Abad and Kian (2014) who point to the lack of connection between the content of the primary school math curriculum and the environment around the students as a challenge of the primary school curriculum. The research result of Alajmi (2012) also shows that Japanese mathematics textbooks in presenting the concept of fractions are focused on relating the subject to the real life of students, and for this, they get help from word problems that often deal with the topic of measurement, and it is in line with the result of the current research.

The differences seen in presenting the concept of fractions in the reviewed textbooks from the two countries are divided into two categories. The first category belongs to the topics that there are in the second to fourth grades of one country, and it is not among the educational goals in presenting this concept up to the fourth grade for other countries. For example, Iranian students up to the fourth grade become familiar with discrete representations of fractions, simplifying fractions, multiplying integers by fractions, comparing fractions with equal denominators, the relation between probability and fractions, and showing approximate fractions for shapes. However, these issues and concepts of fractions have not been addressed for the second to fourth grades Japanese students. Meanwhile, Japanese students up to fourth grade learn how to convert greater than unit fraction into a mixed number and contrariwise, and they know how to add and subtract them throughout the subject of operation on fractions; while these two topics are not among the goals of presenting the concept of fractions in the mathematics curriculum of Iran until the end of the fourth grade. With a closer look at the topics presented in the concept of fractions, we will come to topics that students in both countries will be familiar with until the end of the fourth grade of primary school, but these topics present in different grades for students in both countries.

The second category refers to the differences. For example, familiarity and understanding of a part of a whole in the continuous representation of the subconstructs part-whole are in the goals of the second grade mathematics textbook of Iran, but Japanese students in the second grade are familiar with a part of a whole that is divided into two, four and eight parts equally, and in the third grade they develop the subject. As mentioned earlier, the issue of comparing fractions with equal numerators is one of the common subjects in the concept of fractions for both countries, but it has been presented in different grades. As Iranian students learn this skill in the third grade, Japanese students learn it in the fourth grade. In addition, one of the important topics about the concept of fractions in primary school is writing equal fractions that present to Iranian students in the third grade mathematics textbook and for Japanese students during their fourth grade. The purpose of the present research is in line with Tajari (2012) due to the importance of paying attention to the concept of fractions in the textbooks of different countries.

One of the reasons that show the necessity of conducting this research is the challenge of Iranian students in dealing with problems related to fractions in TIMSS, and on the other hand, the performance of Japanese students is higher than the international average in dealing with problems presented from this concept in TIMSS. (Bakhshalizade& Kashefi, 2017). The results of the present study showed that Iranian second to fourth grades mathematics textbooks have a higher percentage of pages on the concept of fractions than the reviewed Japanese mathematics textbooks,

and Iranian textbooks emphasize the concept of fractions more than Japanese textbooks. But the results of TIMSS show that the performance of Iranian students compared to Japanese students in the common and different topics of fractions was weaker. The lower performance of Iran in the concept of fractions compared to Japan shows that in the common subjects between two countries, only more pages and problems in the textbook will not help, but the quality of presentation of concepts, the connection of the concept of fractions with other mathematical concepts and the previous experiences of the learners are also of great importance, and it is necessary to review and revise Iran's mathematics textbooks in common subjects with Japan on the concept of fractions and authors should take appropriate steps to improve the textbooks in this concept.

Another important point that can be mentioned is that with the changes in our country's textbooks in 2010, the Iranian fourth grade students who participated in TIMSS (2015, 2019) were a generation of students who were affected by the curriculum changes and structural changes in the educational system. So getting familiar with the common and non-common topics of fractions and further investigations on the performance results of Iranian students in the problems published from TIMSS 2015 as well as TIMSS 2019 and comparing them with the international and Japanese averages can help the authors and mathematics education experts to understand the effects of the changes made, both positive and negative, on students' mathematical progress. Authors of textbooks can review and modify mathematics textbooks in the subject of fractions by considering this issue and other factors that can affect the results and mathematical performance of students and create a change in the mathematical performance of students in this topic. This issue has been raised in other studies as well. As an example, we can mention the studies of Pejman and Gooya (2018) and Zianejad Shirazi (2015). Local researchers believe that the changes in textbooks and the educational system in Iran have been accelerated, and as a result, with its rapid implementation on a national scale, the evaluation of the new curriculum was not done with deliberation and also after the changes in the textbooks, the teachers did not prepare. So, more time was needed to implement it and prepare teachers for the classrooms (Pejman & Gooya, 2018; Rafie Pour, 2021).

It is worth mentioning that textbooks should not be considered the only effective factor in students' learning results and many other things besides textbooks can be a mediator between the intended and implemented curriculum and affect students' mathematical performance. Researchers (Jafari Kafi Abad & Kian, 2014; Firouzshahi & Yaftian, 2022; Kiamanesh & Mohsenpour, 2010; Chávez, Tarr, Grouws & Soria, 2015; Remillard, 2005; Van Steenbrugge, Valcke & Desoete, 2013; Wijaya, 2017) to many mediating factors. including organizational contexts and politics, norms, teachers' beliefs, orientations, students' attitudes towards mathematics, how to prepare teachers

and their educational background, the lack of useful and sufficient in-service courses for teacher training, the extent of using textbooks while teaching and the structure of classrooms have mentioned. The effect of these factors along with textbooks on students' learning results requires more research. In the research of Kian et al. (2019), they believe that factors outside the textbook, such as changes in textbooks, weak and insufficient courses for teacher preparation, and traditional methods of teaching mathematics can also effect on the poor performance of students, all of which require more research. It is suggested to those who are interested in research in this field that it is possible to investigate the effect of textbooks along with the factors that were mentioned on the results of students' mathematical performance, especially in the concept of fractions.

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