Analyzing the Reasons for the Differences in Geometric Performance of Fourth-Grade Iranian and Japanese Students in Released Problems of TIMSS 2019 with an Emphasis on the School Textbook

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ARTICLE INFO	ABSTRACT
	This is a comparative research aimed at finding the reasons for difference in the geometric performance of fourth-grade students in Iran and Japan in TIMSS 2019 with an emphasis on school textbooks. For this purpose, the percentage of correct responses from students in both countries to the released measurement and geometry problems in TIMSS 2019, the timing of the introduction of geometric topics, and similar problems in the textbooks of both countries were examined. The strategy for selecting countries was "different social systems, different educational outcomes", and the data collection method was documentary. Qualitative analysis was employed for analyze of data. The findings revealed that in all seven topics of the Measurement and Geometry domain in TIMSS 2019, Iran's average percentage was lower than Japan's, and Japan's average correct responses were higher in all problems except for one problem on symmetry. Also, differences were observed in the focus, timing of introduction, and vertical connections of geometric topics in the school textbooks of the two countries. Furthermore, the problems in Japanese textbooks exhibited greater diversity, with varied perspectives in problem design. Iran performed better in solving problems that had nearly identical examples in the textbook, while Japan demonstrated high performance even in problems without similar examples. Based on the findings, it is suggested that Iranian educational planners incorporate problems with varied cognitive levels in textbook content and consider diversity in problem design for each topic.

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1. Introduction

International comparative studies such as TIMSS (Trends in International Mathematics and Science Study), which examine the mathematics and science achievement trends of fourth and eighth-grade students across multiple countries, have been conducted every four years since 1995 by the International Association for the Evaluation of Educational Achievement (IEA). This study evaluates mathematics and science in two domains: the 'content' domain- including Numbers, Measurement and Geometry, and Data- and the 'cognitive' domain- encompassing knowing, applying, and reasoning. In TIMSS 2019, the Measurement and Geometry content domain accounted for approximately 30% of the fourth-grade mathematics items. This domain represents one of the most challenging areas for mathematics teaching and learning. Due to the consistently low performance of students across various countries in this domain (Mullis et al., 2012, 2020), it has attracted significant attention from mathematics education researchers, curriculum reformers, textbook authors, and teachers (Shokatajnonna, 2021; Abdullah & Shin, 2020; Noreen & Rana, 2019; Fan et al., 2018; Fujita, 2012).

Given the absence of a national organization or center for educational assessment in Iran's educational system, the findings from TIMSS can be utilized to evaluate and track the performance trends of Iranian students in the content domain of Measurement and Geometry (Kiamanesh, 2022). An examination and tracking of fourth-grade Iranian students' results in solving geometry problems from TIMSS 2019 reveal that Iran has demonstrated below-average performance compared to the international mean in this content domain, while Japan has achieved a respectable mean score and ranking among participating countries (Mullis et al., 2020). Table 1 presents the average performance of Iran and Japan overall, as well as across the three content domains based on TIMSS 2019 results.

Table 1: Mean scores of Iran and Japan in fourth-grade mathematics

Average Performance / Country	Overall	Domain		
		Number	*Measurement and Geometry*	Data
Iran	443	446	445	424
Japan	593	586	601	606
difference	150	140	156	182

Table 1 reveals that Iran trails Japan by 150 score points in overall mean performance, with this performance gap being particularly pronounced in the Measurement and Geometry content domain (Mullis et al., 2020). The disparity between Iran and Japan in this specific domain reaches 156 score points - a difference that exceeds the overall mean gap by 6 points. Furthermore, TIMSS 2019 results indicate that only 4% of Iranian students answered Measurement and Geometry items correctly at the lowest proficiency level, compared to 37% of Japanese students at the same level - an observed difference worthy of serious consideration. This performance gap is not unique to TIMSS 2019; previous TIMSS cycles have similarly shown Iranian students underperforming their Japanese counterparts in the Measurement and Geometry content domain (Table 2).

Table 2: Performance of Iran and Japan in the Measurement and Geometry content domain – TIMSS fourth-grade

Country/Year	2007	2011	2015	2019
Iran	408	435	428	445
Japan	575	589	601	601
difference	167	154	173	156

As shown in Table 2, the largest score difference in this content domain was observed in 2015. In 2019, with a score difference of 156 points, the performance gap between the two countries decreased compared to the previous cycle, yet a substantial difference remains (Mullis et al., 2020).

Teachers' instructional methods in classrooms and their assessment approaches can significantly influence students' performance in both countries during international studies. Official documents from Iran (Reyhani, 2016) and Japan (Ministry of Education, Culture, Sports, Science and Technology, 2020) reveal that both countries' curricula emphasize interactive and active learning approaches, as well as competency-based education. The documents from both nations also highlight process-oriented assessment rather than sole focus on final outcomes, encouraging the use of evaluation methods that measure student progress over time. Another common feature is the implementation of performance-based assessment, where students must demonstrate their abilities in authentic, practical situations. At the primary level, this may include hands-on projects, oral presentations, and participation in group activities. Consequently, given the observed similarities in teaching and assessment methods according to both countries' curricular documents, the current study focuses on the implemented curriculum content and textbooks, as examining these elements is crucial for understanding the learning opportunities provided to students and the

reasons behind their performance differences (Charalambous et al., 2010; Van den Ham & Heinze, 2018; Yaghoobi et al., 2022).

Another approach used to understand differences in education and achievement among different nations in international studies, such as TIMSS, is the analysis of learning opportunities, including curriculum content and textbooks (Choi & Park, 2013; wijaya, 2017). Curriculum content refers to the topics taught to students. In TIMSS, this learning opportunity is assessed through a questionnaire completed by teachers and the coordinating team in each participating country, focusing on various mathematics topics and the timing of their instruction. The questionnaire examines three content domains from the curriculum and textbooks of each participating country: Number, Measurement and Geometry, and Data. The Number and the Measurement and Geometry domains are each subdivided into seven specific topics, while the Data domain is categorized into three topics. For each domain, teachers from different countries provide information regarding the grade level at which students are first introduced to each of these topics.

School Textbooks, as another key learning opportunity, can be analyzed by examining content coverage, structure, sequencing of content, and problem presentation. Numerous studies investigating the relationship between textbook content and student achievement in international studies like TIMSS have consistently confirmed the significant impact of textbooks on student performance (Hadar, 2017; Van den Ham & Heinze, 2018).

Since different learning opportunities lead to different outcomes, and the mathematics curriculum along with its primary product- mathematics textbooks- plays a crucial role in creating these opportunities (Hadar, 2017; Van den Ham & Heinze, 2018), developing high-quality mathematics textbooks has become a critical concern for textbook authors worldwide. Given the importance of Measurement and Geometry, many researchers outside Iran have focused on comparative studies of mathematics textbooks (Takeuchi & Shinno, 2020; Abdullah & Shin, 2019; Fan et al., 2018; Miyakawa, 2017; Fan et al., 2016; Wang & Yang, 2016; Choi & Park, 2013; Fujita & Jones, 2014; Jones & Fujita, 2014).

Considering the pivotal role of curriculum and textbooks in shaping learning opportunities and their direct impact on student achievement, literature emphasizes that low-performing countries in studies like TIMSS should conduct comparative research focusing on curriculum and textbook analysis, particularly in Measurement and Geometry. This approach can help identify reasons for performance differences across countries and inform educational reforms (Hidayah & Forgasz, 2020; Abdullah & Shin, 2019; Bokhove et al., 2019; Fan et al., 2018). Although Iranian studies have compared mathematics curricula and textbooks with other countries or TIMSS

frameworks (Kiamanesh, 2022; Firouzshahi & Yaftian, 2022; Kian et al., 2019; Rezasoltani et al., 2019; Azadi & Shirvani Shiri, 2019; Pejman & Gooya, 2018; Danaei Zarchi, 2017; Izadi et al., 2015; Karami Zarandi, 2009), most have focused on general mathematics domains and have paid less attention to comparative analyses specifically targeting the Measurement and Geometry content areas in the textbooks of different countries, as well as the problems presented within them. This research gap highlights the need for deeper investigations to identify the reasons behind the strengths and weaknesses in students' geometric achievement. Given this need, along with the importance of Measurement and Geometry in curricula, Iranian students' low-performance in this domain, and the connections between geometry to other disciplines, conducting a comparative study in this field becomes imperative. Since the researchers' literature review did not identify any study specifically examining and comparing the Measurement and Geometry domains in Iran's primary mathematics curriculum and textbooks with those of high-performing countries (such as Japan) in international studies, the present study aims to address this gap and provide novel insights for enhancing the mathematical performance of Iranian students. Specifically, this study examines the difference in performance between Iranian and Japanese fourth-grade students on the released TIMSS 2019 problems in the domains of Measurement and Geometry. Additionally, in order to identify some of the reasons for these differences, the degree of emphasis and attention given to Measurement and Geometry topics in the two countries' textbooks, along with the manner of presenting problems related to this domain, has been analyzed. This research answers the following question:

- What are the reasons for the difference in geometric performance between Iranian and Japanese fourth-grade students on the released TIMSS 2019 problems, with emphasis on how geometric problems are presented in the two countries' textbooks?

2. Research Method

The method of study was comparative, and the statistical population was the countries participating in the TIMSS 2019. The country selection strategy was "different social systems, different educational outputs". Unlike Iran, Japan utilizes various textbooks aligned with the national curriculum for teaching in schools, and these two countries have two different performance levels in the Measurement and Geometry content domain in TIMSS. The problems in the Measurement and Geometry domain in TIMSS 2019 consist of 52 problems. Since only a limited number of items are released in each TIMSS cycle, and the rest are kept unreleased, and in TIMSS

2019, 23 items were released, the data examined in this study consisted of the 23 released items in the Measurement and Geometry domain (Bakhshalizade & Kashefi, in press). In this study, the data collection method was documentary, conducted through the examination of primary sources such as official and governmental documents and mathematics textbooks of the two countries, as well as secondary sources such as articles and other related books. The data collection instrument was also content analysis forms, the face and content validity of which were confirmed by three mathematics education professors from Shahid Rajaee Teacher Training University and Farhangian University, as well as three experienced primary mathematics teachers from Tehran Province. Additionally, the reliability of this instrument was determined using Cohen's kappa coefficient, for which two coders familiar with the subject examined and coded the problems in the mathematics textbooks of Iran and Japan based on the released problems from the TIMSS 2019. The inter-rater agreement coefficient obtained for coding the problems in Iranian mathematics textbooks was 0.868, and for coding the problems in Japanese mathematics textbooks, it was 0.839.

To identify some of the reasons for the difference in geometric performance of students in these two countries in TIMSS 2019, the framework presented in Wijaya's study (2017), which is related to examining the content objectives of the Measurement and Geometry domain and the timing of these topics based on the TIMSS questionnaire and comparing the problems presented in the textbooks of countries, was used. Similar to Wijaya's study (2017, pp. 224-225), first, the percentages of correct responses by students to TIMSS problems were compared and analyzed, for which descriptive statistics were used. Then, for each country separately, the responses of the national TIMSS organizing team to a questionnaire regarding the timing of instruction of various geometry topics were examined and compared to identify potential reasons for the high or low performance of students in the two countries in solving problems. For this purpose and to obtain a clearer picture of the reasons for the difference in geometric performance between the two countries, the textbooks and the problems presented in them were qualitatively compared based on the problems released in TIMSS.

It is noteworthy that students are more likely to succeed in solving problems for which they have previously seen a familiar example, but they may perform less effectively in new problem-solving situations. The familiarity that students gain with problems and their solutions will help them transfer to a problem with a new situation (Choi & Park, 2013). Therefore, examining the released TIMSS problems based on finding similar examples in textbooks can help answer the question of whether students' high performance in problems is influenced by exposure to these problems in textbooks and what performance they demonstrate when they have not encountered a

problem similar to the TIMSS problem in the textbook. It should be noted that in the textbooks of both countries, problems considered similar to the released TIMSS problems were those that were aligned in terms of objectives with the expected goals in TIMSS problems and required the application of the same geometric skills and concepts.

An examination of the structure of the TIMSS 2019 questionnaires showed that the "Measurement and Geometry" content domain is categorized into seven topics. Accordingly, in the data analysis, each of the 23 selected problems from the TIMSS 2019 was classified into one of these seven topics based on their content. The topics of the Measurement and Geometry domain in TIMSS 2019 are as shown in Table 3.

Table 3: TIMSS 2019 questionnaire regarding the content domain of Measurement and Geometry

	Mostly taught before this year	Mostly taught this year	Not yet taught or just introduced
Measurement and Geometry			
1) Solving problems involving length, including measuring and estimating	○	○	○
2) Solving problems involving mass, volume, and time.................	○	○	○
3) Finding and estimating perimeter, area, and volume..................	○	○	○
4) Parallel and perpendicular lines...	○	○	○
5) Comparing and drawing angles...	○	○	○
6) Elementary properties of common geometric shapes.................	○	○	○
7) Three-dimensional shapes, including relationships with their two-dimensional representations..	○	○	○

Source : Mullis, et al (2020)

Through completing this questionnaire and responding to these three options, teachers have indicated whether these topics were "taught to students before fourth-grade", "taught in fourth-grade", or "not taught or only introduced to students". The information provided on the TIMSS website indicates that the extent of topic coverage varies among participating countries. According to data reported by Iranian and Japanese teachers in the TIMSS 2019, Iranian students had been

taught 74% of geometry topics by fourth-grade, while this percentage was 81% for Japanese students (Mullis et al., 2020).

After determining which of the seven subdomains of Measurement and Geometry each released problem belonged to, the percentage of correct responses for Iran and Japan on each problem was determined, and using this, the average percentage of correct responses for each topic was calculated. Subsequently, the content objectives and topics in the Measurement and Geometry domain in Iran and Japan were examined, and the teaching and learning expectations in this domain were specified for each country. To further verify this matter, reference was made to a questionnaire containing responses from the TIMSS National Research Coordinators regarding whether each geometry topic was included in the educational curriculum and textbooks of the two countries. According to this questionnaire, the level of attention and emphasis on each geometry topic and the vertical connections of each topic- the sequence of related concepts and learning skills across different grade levels- were compared between the textbooks of the two countries. Then, four problems where Iran showed the largest performance gap with Japan and four problems with the smallest performance gap were considered. These eight problems were selected as the basis for searching in the textbooks of grades 1 to 4 in both countries, and similar problems to these eight in terms of content and solution methods, if any, were extracted from the textbooks and compared with each other. The purpose of this comparison was to identify existing differences in the presentation of problems in the textbooks of Iran and Japan, particularly in areas where the performance gap between students of the two countries was reported to be small or large. The coding of textbook problems benefited from the opinions and guidance of mathematics education specialists.

It should be noted that compulsory education in Japan lasts for 9 years, and primary education includes grades 1 to 6 (ages 6 to 12). The compilation of Japanese textbooks is carried out according to the instructions of the Ministry of Education and based on the national curriculum, which has resulted in the preparation and development of six different textbook series based on this national curriculum for primary education (Takahashi & Shinno, 2020). Each of these series is published by different companies and publishers, and local educational boards are responsible for selecting school textbooks. In the current research, Tokyo Shoseki publications have been used for Japanese primary mathematics textbooks, which are the most widely used in Japanese schools (Alajmi, 2012; Isoda & Olfos, 2021; Takahashi, 2016; Fujii, 2018; Watanabe et al., 2017). Their English translations are also available, and a review of comparative studies conducted on Japan shows that in numerous studies, the Tokyo Shoseki textbook series for primary education have

been used as one of the main sources (Alajmi, 2012; Isoda & Olfos, 2021; Takahashi, 2016; Takahashi & Shinno, 2020). Similar to Japan, Iran also has 9 years of compulsory education, and primary education includes grades 1 to 6. However, unlike Japan, in Iran, according to the instructions of the Ministry of Education, one series of textbooks is prepared and developed for each grade level, and therefore, in primary education, one unified mathematics textbook is published nationally for each grade, and all are required to use it.

3. Findings

Table 4 categorizes all 23 released TIMSS 2019 items from the "Measurement and Geometry" content domain according to the seven geometry topics outlined in the TIMSS questionnaire. It also presents the percentage of correct responses by students from both countries for each item, as well as their average correct response rate for each topic.

Table 4: Released Measurement and Geometry items, categorized by TIMSS 2019 geometry topics

Geometry topics in TIMSS 2019	problem Code	Cognitive Domains	Content of each problem	Average percentage of correct answers		
				Iran	Japan	International
Solving problems involving length, including measuring and estimating	MP02-06	knowing	The appropriate unit for each measurement	42	83	60
	MP06-08	knowing	Calculate the length of the line segment in centimeters	35	75	46
	MP07-07	knowing	Calculate the length of the wire in centimeters	19	64	32
Average percentage of correct answers in the topic				**32**	**74**	**46**
Solving problems involving mass, volume, and time	MP02-07	Reasoning	Calculating the weight of a pear	40	73	57
	MP05-04	Applying	Calculating the time it takes to finish baking a cake	31	81	54
	MP05-11	Reasoning	Calculating the number of books to fill the box	22	81	46
Average percentage of correct answers in the topic				**31**	**78/3**	**52/3**
Finding and estimating perimeter, area, and volume	MP02-08	Applying	Number of shapes needed to fill the square surface	11	41	23
	MP05-10	Applying	Calculating the length of a triangle's side and its perimeter	33	37	42
	MP07-10	Applying	Calculating the area of a rectangle	42	81	51
Average percentage of correct answers in the topic				**28/6**	**53**	**38/6**

Parallel and perpendicular lines	MP01-06A	Applying	Identifying parallel streets	41	85	47
	MP01-06 B	knowing	Identifying perpendicular streets	63	88	73
	MP03-08	knowing	Identifying parallel sides in a trapezoid	26	69	35
Average percentage of correct answers in the topic				**43/3**	**80/6**	**51/6**
Comparing and drawing angles	MP05-08	knowing	Shape with one right angle	72	79	64
	MP06-09	knowing	Angles smaller than a right angle in the shape	34	78	45
	MP07-08	Applying	Drawing an angle smaller than a right angle	39	55	43
Average percentage of correct answers in the topic				**48/3**	**70/6**	**50/6**
Elementary properties of common geometric shapes	MP02-09	Applying	Completing the shape based on the line of symmetry	68	56	77
	MP05-07	Reasoning	Identifying the classification rule for shapes	23	51	30
	MP05-09	knowing	Shape with axial and rotational symmetry	47	49	52
Average percentage of correct answers in the topic				**46**	**52**	**53**
Three dimensional shapes, including relationships with their two-dimensional representations	MP06-10	Applying	Number of triangles and squares for constructing 3D shapes	4	47	26
	MP01-08	Reasoning	Identifying the box from its net	16	40	30
	MP01-07	knowing	Identifying the number of triangles needed to build a triangular pyramid	46	77	70
	MP03-10	Reasoning	Identifying the net of a decorated cube	40	71	59
	MP07-09	Reasoning	Identifying the net of a given shape	53	86	63
Average percentage of correct answers in the topic				**31/8**	**64/2**	**49/6**

As shown in Table 4, out of the 23 items, three items correspond to each of topics 1 through 6, while five items are dedicated to topic 7, titled "Three-dimensional shapes, including relationships with their two-dimensional representations". Furthermore, in each topic, the performance of Iranian students was lower than that of their Japanese counterparts. This performance gap was significant in all topics except for the "Elementary properties of common geometric shapes". Additionally, Iranian students achieved their highest performance in the "Comparing and drawing angles" items, with an average correct response rate of 48.3%. Conversely, Japanese students showed their best performance in the topic of "Parallel and

perpendicular lines", with an average correct response rate of 80.6%. For Japan, the average correct response rate for items on "Finding and estimating perimeter, area, and volume" (53%) and for items on "Elementary properties of common geometric shapes" (52%) were significantly lower than their average across the remaining topics. Similarly, Iranian students had their lowest performance in "Finding and estimating perimeter, area, and volume" items, with an average correct response rate of 28.6%.

According to Table 4, the highest percentage of correct responses from Iranian students was 72%, achieved on item MP05-08, which belongs to the cognitive domain of "Knowing". In contrast, their lowest performance was 4%, on item MP06-10 from the cognitive domain of "Applying". The lowest percentage for Japanese students was 37% (item MP05-10), which is notably higher compared to the lowest percentage for Iranian students (4%). With the exception of item MP02-09, Iran's performance was lower than Japan's on all items. On two items (MP05-08 and MP05-09), Iran's performance was close to that of Japan. For the remaining items, regardless of their associated cognitive domain, the performance gap was substantial. To investigate the reasons for the low performance of Iranian students in the Measurement and Geometry content domain, the mathematics curricula of both countries for this domain were examined using the TIMSS website. The mathematical topics covered in this domain by the end of fourth-grade in Iran are as follows:

- Metric units;
- Measuring, estimating, and drawing and comparing angles;
- Length;
- Areas of common two-dimensional shapes;
- Parallel and perpendicular lines;
- Properties of common geometric shapes;
- Calculating the perimeter and area of parallelograms, rectangles, triangles, and squares;
- Introduction to shapes with lines of symmetry (Mullis et al., 2020).

The topics in the Measurement and Geometry domain for Japan by the end of fourth-grade are listed as:

- Understanding units of area measurement and using calculations to determine the area of geometric shapes;
- Understanding the meaning of units and measurements for angles and measure angles;
- Understanding plane figures (e.g., parallelograms, rhombuses) and spatial figures (e.g., rectangular parallelepipeds) by observing their elements and exploring the relationships between those elements;
- Recognizing the elements and positional relationships of two- and three-dimensional geometric shapes through activities such as observing and drawing these shapes (Mullis et al., 2020).

An examination of these stated objectives and topics related to the Measurement and Geometry domain from both countries, as presented on the TIMSS website, cannot by itself guarantee their full implementation in the classroom. Furthermore, given the observed similarities in teaching and assessment approaches between the two countries, the research also referred to the responses provided by the National Research Coordinators of TIMSS 2019 in each country regarding the intended grade level for introducing various topics in this domain (Mullis et al., 2020). The timing of topic introduction and the vertical coherence between them were compared for both countries to identify potential reasons for the differences in students' geometric performance (see Table 5).

Table 5: Vertical coherence of TIMSS 2019 geometry topics and their intended grade of introduction in the mathematics textbooks of Iran and Japan

Grade / Topic		First	Second	Third	Fourth	Average percentage
Solving problems involving length, including measuring and estimating	Iran	✓	✓	✓	✓	32
	Japan	✓	✓	✓	-	74
Solving problems involving mass, volume, and time	Iran	✓	✓	✓	✓	31
	Japan	✓	✓	✓	-	78/3
Finding and estimating perimeter, area, and volume	Iran	-	✓	✓	✓	28/6
	Japan	-	-	-	✓	53
Parallel and perpendicular lines	Iran	-	-	-	✓	43/3
	Japan	-	✓	✓	✓	80/6
Comparing and drawing angles	Iran	-	-	✓	✓	48/3
	Japan	-	-	✓	✓	70/6
Elementary properties of common geometric shapes	Iran	✓	✓	✓	✓	46
	Japan	-	✓	-	-	52
Three-dimensional shapes, including relationships with their two-dimensional representations	Iran	-	-	✓	✓	31/8
	Japan	–	✓	–	✓	64/2

Based on Table 5, certain topics in Iran's mathematics textbooks, such as the first, second, third, and sixth topics, demonstrate vertical coherence. Students in Iran have been continuously exposed to these specific subjects up to the fourth-grade. Regarding the fifth and seventh topics, Iranian students have engaged with the relevant material across two different grade levels.

However, the fourth topic, "Parallel and perpendicular lines", is only covered in a single grade- the fourth-grade - and unlike the other topics, it is not revisited or practiced across multiple grade levels. In the case of Japan, the first, second, and fourth topics also exhibit vertical coherence, being introduced in the curriculum for at least three consecutive grades. The fifth topic, "Comparing and drawing angles", is included in the third and fourth-grade curricula, while the seventh topic on "Three-dimensional shapes" is covered in the second and fourth-grades. In contrast, the topics "Finding and estimating perimeter, area, and volume" and "Elementary properties of common geometric shapes" are each addressed in only a single grade level within the Japanese curriculum.

Furthermore, Table 5 illustrates the extent of vertical coherence for each topic and suggests some potential reasons for the students' level of success. Japanese students were consistently exposed to the concept of "Parallel and perpendicular lines" across the second, third, and fourth-grades. This sustained and sequential coverage in the textbook likely contributed to their higher average correct response rate for this topic. On the other hand, they encountered the topic of "Elementary properties of common geometric shapes" only in the second grade and did not achieve a satisfactory average correct response rate for it compared to other topics. A same situation applies to the topic "Finding and estimating perimeter, area, and volume". According to the view of Japanese National Research Coordinator for TIMSS, Japanese students are only introduced to this topic in the fourth-grade. Perhaps presenting this topic in the fourth-grade, simultaneously with the administration of TIMSS at this grade level, deprived the students of the opportunity to review and practice the topic. As a result, Japanese students, with an average correct response rate of 52%, did not perform as satisfactorily in this topic compared to other topics.

For Topics 1 and 2, despite their inclusion across multiple grades and adherence to a spiral curriculum, Iranian students, unlike their Japanese counterparts, did not achieve high performance on the related items. Regarding Topic 3, "Finding and estimating perimeter, area, and volume", Iranian students were exposed to this topic repeatedly across three grades. In contrast, the Japanese curriculum introduces it only in the fourth-grade. Nevertheless, the average correct response rate reveals that Iranian students performed substantially lower than Japanese students on this topic. Topic 6, "Elementary properties of common geometric shapes", was covered in Japan in a single grade, whereas it was addressed from the first to the fourth-grade in Iran. Although the emphasis on this topic in Iran was similar to that on Topics 1 and 2, it appears that Iranian students possessed a better command and higher mastery in responding to items in this specific area. Topics Five and Seven are covered in two grade levels in both Iran and Japan; however, the average correct

response for these two topics indicates a high performance disparity between Iranian and Japanese students.

Subsequently, to identify a potential explanation for the low performance of Iranian students in the Measurement and Geometry domain and its individual topics, the problems presented in the two countries' textbooks were also examined. This analysis aimed to determine, following the review of the timing of topic introduction and their vertical coherence from first to fourth-grade in both countries, whether Iranian students performed poorly due to the unfamiliar format of TIMSS problems, or if they faced difficulties in responding despite encountering similar examples in their own textbooks. Furthermore, a comparison with Japanese textbooks was conducted to uncover potential reasons for the higher performance of Japanese students compared to their Iranian counterparts. An analysis of the difference in the percentage of correct responses between the two countries on the released Measurement and Geometry problems revealed that the performance gap was substantial in some items and smaller in others. Due to the large number of items, Table 6 presents four items with the largest performance gap between Iranian and Japanese students, followed by four items with the smallest performance gap.

Table 6: Items with the highest and lowest performance gaps in Measurement and Geometry domain for Iran and Japan

Problem code	Percentage of correct answers		Difference in percentage of correct answers between Japan and Iran	Cognitive domain
	Iran	Japan		
MP02-09	68	56	-12	Applying
MP05-09	47	49	2	Knowing
MP05-10	33	37	4	Applying
MP05-08	72	79	7	Knowing
MP06-09	34	78	44	Knowing
MP07-07	19	64	45	Knowing
MP05-04	31	81	50	Applying
MP05-11	22	81	59	Reasoning

Item MP02-09 was the only problem in which Iranian students performed better than Japanese students, with a difference of 12 percentage points. For three items (MP05-09, MP05-10, and MP05-08), Iran's performance was close to Japan's, with the performance gap between students from the two countries being in the single digits. For the remaining items, the difference in correct response rates was considerable. According to Table 6, on item MP05-11, Iranian students

trailed Japanese students by a significant margin of 59 percentage points. To identify differences in how mathematical problems are presented in the two countries' textbooks, the items in Table 6 served as the basis for examining these textbooks. In what follows, these TIMSS items are presented alongside similar sample problems from the textbooks, following the order presented in Table 6. The first item reviewed is item MP02-09 from the cognitive domain "Applying". This is the only item where Iranian students outperformed their Japanese counterparts. Figure 1 presents this TIMSS 2019 item along with a similar sample problem from the Iranian second-grade mathematics textbook.

Figure 1: TIMSS 2019 released item MP02-09 and a similar sample
from the Iranian second-grade textbook

Table 5, along with the analysis of the two countries' geometry textbooks, reveals that Iranian students encounter the topic of basic properties of geometric shapes in their textbooks from grades 1 to 4. They practice using fundamental properties, such as symmetry, to describe, compare, and draw common two-dimensional shapes in each grade. In this problem from the Iranian second-grade mathematics textbook, students are also asked to complete the other half of given shapes based on the line of symmetry. A notable point is that the solved example in the textbook is similar to the TIMSS item.

According to Table 5, unlike their Iranian peers, Japanese students are introduced to some properties of common geometric shapes only in the second grade. The examination of Japanese textbooks revealed that the topic of symmetry and problems similar to this TIMSS item are not present in the reviewed textbooks from this country. In contrast to Japan, the sustained emphasis on the topic of symmetry in the Iranian curriculum and textbooks, coupled with the presence of sample problems similar to the TIMSS item, has enabled Iranian students to outperform their Japanese counterparts in this topic. In another released TIMSS item, coded MP05-09 from the

cognitive domain "Knowing", the performance difference between the students of the two countries was very close, with a mere 2 percentage point gap. This item is presented in Figure 2.
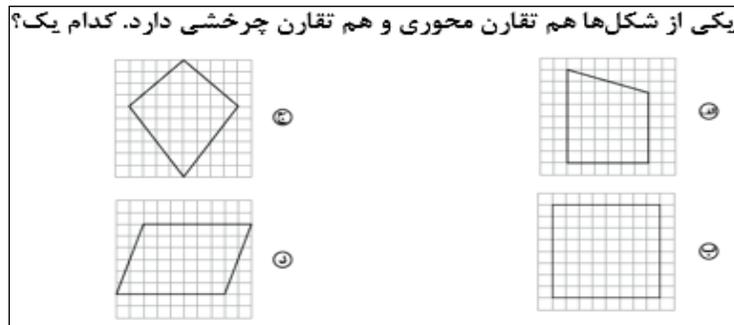

Figure 2: TIMSS 2019 released item MP05-09

Tracing this item through the two countries' textbooks revealed that the topic of axial and rotational symmetry, along with sample problems, is not covered in either Iranian or Japanese textbooks by the fourth-grade. Nevertheless, Japanese students managed to perform better than Iranian students on this item.

The next item, coded MP05-10 from the cognitive domain "Applying", shows a close performance between Iranian and Japanese students, with a performance gap of only 4 percentage points. This item is displayed in Figure 3.
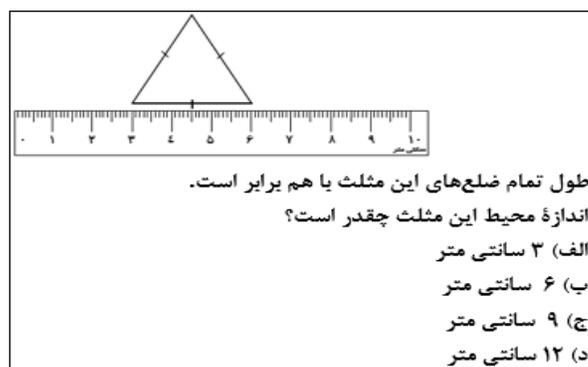
Figure 3: TIMSS 2019 released item MP05-10

The first part of the problem involved length measurement. The length measurement problems in Iranian mathematics textbooks exclusively asked students to calculate lengths where

the starting point was aligned with the zero mark on the ruler. Therefore, calculating lengths like the one in the problem above may have been challenging for Iranian students. In contrast, Japanese textbooks present situations similar to the TIMSS item for calculating length. The second part of the problem related to the concept of perimeter and calculating the perimeter of an equilateral triangle. This concept is introduced in Iran's third-grade textbook, and students practice with sample problems requiring perimeter calculation. An examination of Japanese mathematics textbooks revealed that calculating the length around a shape is included from the second grade, and the fourth-grade curriculum covers how to calculate the perimeter and area of various geometric shapes. However, no problem exactly identical to the TIMSS item was found in the reviewed textbooks from either country.

In another item, Iranian and Japanese students demonstrated close performance levels, with a 7 percentage point difference in their average scores. On this item (MP05-08), Iranian students achieved a correct response rate of 72%, representing their highest performance across all released items in the Measurement and Geometry domain. Figure 4 presents this TIMSS item along with a similar sample problem from the Iranian third-grade mathematics textbook.
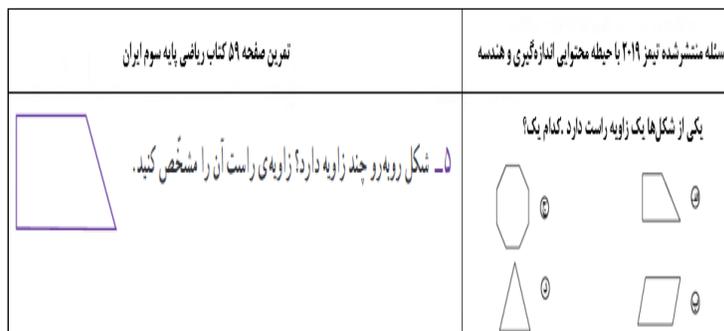

Figure 4: TIMSS 2019 released item MP05-08 and a similar sample from the Iranian third-grade textbook

According to Table 2, the content of this TIMSS item is covered in both countries' curricula in the third and fourth-grades. The examination of Iranian textbooks revealed that a similar example to this TIMSS item is presented on page 59 of the third-grade textbook. Consequently, solving this problem posed little challenge for Iranian students. In contrast, the review of Japanese textbooks showed that no problem directly similar to this TIMSS item is presented. However, various examples and exercises for identifying right, acute, and obtuse angles are included in their materials, indicating that solving this particular problem was also not challenging for Japanese

students. Perhaps the primary reason for Iran's highest performance on this item compared to others can be attributed to the presence of a nearly identical problem in their textbook.

The analysis now turns to TIMSS 2019 items in the Measurement and Geometry domain where a substantial performance gap was observed between Iranian and Japanese students. The largest performance difference in the released TIMSS 2019 items belongs to item MP05-11 from the cognitive domain of "Reasoning". This item is presented in Figure 5.
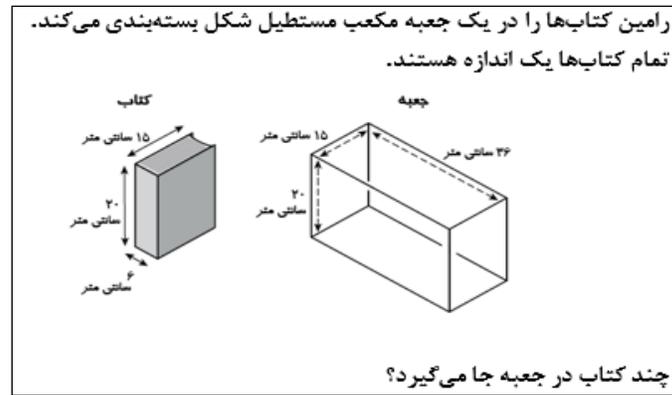
Figure 5: TIMSS 2019 released item MP05-11

The examination of the textbooks from both countries revealed that no problem directly similar to the one presented in TIMSS is included in either curriculum up to the fourth-grade. However, unlike their Iranian counterparts, Japanese students were able to consider the dimensions of both the book and the box, recognize that two of their dimensions were equal, perform the division of 36 by 6, and thus determine that 6 books are needed to fill the box. This indicates the Japanese students' ability to attend to all aspects of the problem and decode it correctly.

Item MP05-04 from the cognitive domain "Applying", with a 50 percentage point performance gap between the two student groups, ranks as the second-largest performance disparity among the released TIMSS items in the Measurement and Geometry domain. This item is shown in Figure 6.

Figure 6: TIMSS 2019 released item MP05-04

Subsequently, Figure 7 presents similar sample problems from the mathematics textbooks of Iran and Japan.

Figure 7: Similar sample problems for item MP05-04 from the Iranian fourth-grade and Japanese third-grade textbooks

The review of Iranian textbooks revealed that in Chapter 4 of the fourth-grade textbook, Iranian students practice sample problems similar to the one posed in TIMSS. In fact, some problems presented in the textbook are even more complex than the TIMSS item. Despite this, Iranian students demonstrated low performance, with a correct response rate of only 31%. Similarly, the examination of Japanese textbooks showed that Japanese books in the second and third grades address problems involving time calculation and elapsed time, providing various sample problems in this area. Japanese students achieved a correct response rate of 81% on this item. Thus, despite the presence of similar sample problems in the textbooks of both countries, the average performance of Iranian students showed a significant gap compared to the correct response rate of their Japanese counterparts.

Another item where a substantial performance gap was observed between the two countries' students is item MP07-07 from the cognitive domain "Knowing". Figure 8 presents this TIMSS 2019 item along with a similar sample from the Japanese second-grade mathematics textbook.
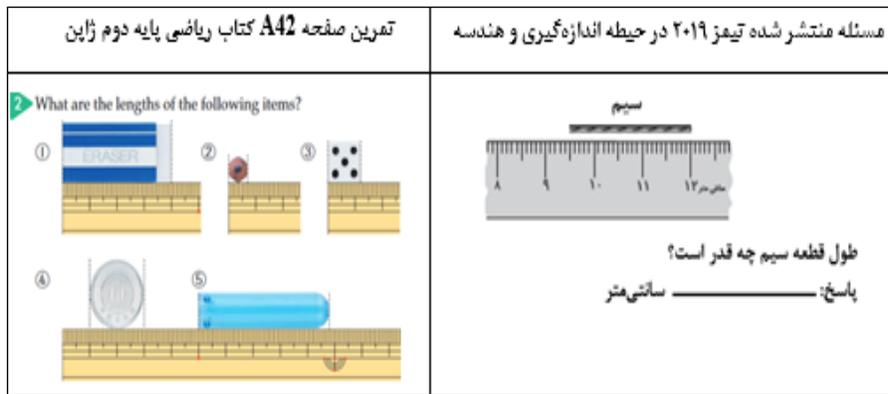
Figure 8: TIMSS 2019 released item MP07-07 and a similar sample from the Japanese second-grade textbook

In the problems presented in Iranian textbooks concerning length measurement and the application of fractions to measure length, all lengths to be measured have their starting point aligned with the zero mark on the ruler. The textbooks do not provide problems that pose a challenge to students in measuring length, nor do they create situations that require students to consider both the starting and ending points of objects for measurement. In contrast, Japanese mathematics textbooks present situations similar to the TIMSS item. Table 5 indicated that Iranian students are exposed to this topic from grades 1 to 4, and Japanese students from grades 1 to 3. Through practice and repetition across consecutive grades, students in both countries have developed sufficient familiarity with the topic. Therefore, the performance difference between Iranian and Japanese students on this item is not solely due to a lack of knowledge of the topic or content. The probable reason may instead lie in the type and quality of the problems presented in the textbooks.

In two other items MP06-09 and MP01-06A, a performance gap of 44 percentage points was observed between the students of the two countries. The first item, from the cognitive domain "Knowing", dealt with comparing and drawing angles. The second item, from the cognitive domain "Applying", pertained to parallel and perpendicular lines. On these two items, the average correct response rates for Iranian students were 34% and 41%, respectively. Since the first item (MP06-09) belongs to the cognitive domain of "Knowing", where students are generally expected to perform better, it was selected as the final case study to investigate the reasons for the low performance of Iranian students. This TIMSS 2019 item is presented in Figure 9.

«الف» ، «ب» ، «ج» و «د» ، زاویه‌های این شکل هستند.

کدام دو زاویه، از زاویهٔ راست کوچک‌تر هستند؟

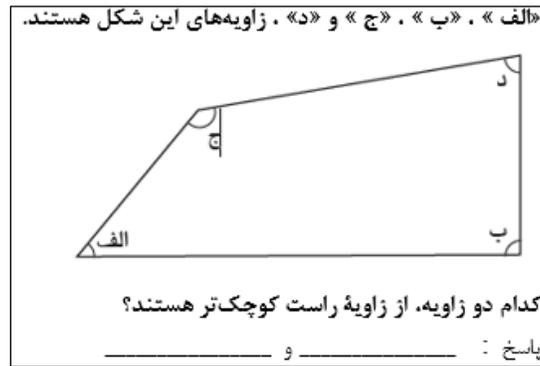پاسخ : ــــــــــــــــ و ــــــــــــــــ

Figure 9: TIMSS 2019 released item MP06-09

An examination of the textbooks from both countries revealed that this particular problem does not have a completely identical counterpart in either set of textbooks. However, this does not mean the underlying concept is absent from the Iranian and Japanese textbook. Both the Iranian and Japanese textbooks include various examples that assess the ability to identify right, acute, and obtuse angles, as well as to compare and draw them. Therefore, solving this problem should not be challenging for students who possess a proper understanding of different angle types. The only potential difficulty lies in the novel formulation of the problem. Consequently, the performance gap in responding to this item is noteworthy. Finally, as a summary of the results, the key similarities and differences identified based on the investigation into potential causes for the disparity in geometric performance between Iranian and Japanese students in TIMSS 2019 are presented below:

Similarities

- Both countries have a national curriculum that serves as the foundation for developing their textbooks.

- Both countries cover very similar content objectives and topics by the end of the fourth-grade.

Differences

- The average percentage of correct responses from Japanese students was higher than that of Iranian students for each geometry topic in TIMSS 2019.

- The average percentage of correct responses from Japanese students was higher than that of Iranian students on all items in the Measurement and Geometry domain, except for one item on symmetry.

- There are differences in the focus, the timing of topic introduction, and the vertical coherence of these topics across different grade levels in the mathematics textbooks of the two countries.

- The problems presented in the Japanese textbooks examined demonstrate greater variety compared to those in the Iranian mathematics textbooks. The design of problems for a given topic in the Japanese materials incorporates different perspectives and contexts.

- Unlike their Japanese counterparts, Iranian students performed well only on those TIMSS items for which they had encountered a very similar example in their textbook. In contrast, Japanese students performed well even in problem-solving situations where no directly similar example was present in their textbook.

4. Conclusion

The findings of the present study indicate that Iranian students have consistently demonstrated lower performance in the Measurement and Geometry domain compared to Japanese students across different cycles of TIMSS. An analysis of the percentage of correct responses for each released item in the TIMSS 2019 Measurement and Geometry domain additionally indicates that Iranian students performed less effectively than their Japanese counterparts in the majority of these items. The only exception was an item requiring the completion of a shape based on a given line of symmetry, where Iranian students managed to achieve a higher performance level than Japanese students. It is noteworthy that the lowest score achieved by Iranian students on the released geometry items was 4 percent, compared to 37 percent for Japanese students. This considerable disparity, especially when considering the lowest performance mark for Iranian students, warrants serious reflection.

The categorization of the released TIMSS 2019 Measurement and Geometry items according to the seven geometry topics revealed that, in each topic, the average percentage of correct answers for Iranian students was lower than that of Japanese students. The highest average performance achieved by Iranian students in these geometry topics was 48.3%, whereas the corresponding average for Japanese students was 80.6%. This notable difference in the average scores is, once again, highly noteworthy.

This performance disparity between the students of the two countries can be examined by considering their learning opportunities, specifically the scope and sequencing of geometry topics in their textbooks- a factor acknowledged by various studies (e.g., wijaya, 2017; Fan et al., 2016;

2018). Since the general curriculum information for the Measurement and Geometry domain provided on the TIMSS website offers only an overview, the present study analyzed the responses from the National Research Coordinators in both countries regarding the intended grade level for introducing each geometry topic. The results for both countries indicated that certain topics in the Iranian and Japanese textbooks are introduced across different grade levels, demonstrating vertical coherence, with their sequence and continuity emphasized over at least three consecutive grades. An analysis of the average correct response rates for students on items related to these topics reveals that Iran, despite following a spiral curriculum and repeating and completing a topic in consecutive grades and paying attention to presenting them at a higher level of complexity and precision in accordance with the cognitive development of students, has not had high performance in responding to TIMSS problems. In contrast, Japan achieved its highest average performance on a topic repeated consecutively from the second to the fourth-grade, and its lowest average performance on a topic covered in only a single grade without reinforcement in other grades. Therefore, based on the information from the reviewed TIMSS questionnaire, the sequencing and structure of geometry topics in Iran's curriculum does not appear to be the primary factor explaining the low performance of Iranian students. To identify other potential reasons for the differences in geometric performance between fourth-grade students in the two countries, the manner in which problems within the Measurement and Geometry domain- particularly those similar to the released TIMSS 2019 items- are presented in the textbooks was examined.

An analysis of the textbooks revealed that for the symmetry item, which falls under the topic "Elementary properties of common geometric shapes", the Iranian curriculum placed significantly greater emphasis on this concept compared to Japan. Iranian mathematics textbooks from grades 1 to 4 included numerous problems on symmetry, and even provided worked examples that closely mirrored the structure of the TIMSS items. Consequently, Iranian students were able to outperform their Japanese counterparts on this particular problem, achieving a correct response rate that approached the international average.

In another item involving identifying a shape with a right angle, under the topic "Comparing and drawing angles", the performance of students from the two countries was similar. The average score of Iranian students on this topic was also higher compared to their performance on other topics. Tracing this problem in the textbooks revealed that a nearly identical example existed in the Iranian textbook. Furthermore, since this item belonged to the "Knowing" cognitive domain and was not highly complex, the presence of a very similar problem in the textbook likely contributed to the high average correct response rate among Iranian students. This finding supports the results of

the study by Choi and Park (2013), who also acknowledged that students are more likely to succeed in solving problems that have similar counterparts in their textbooks. For items with a significant performance gap between Iranian and Japanese students, an examination of the textbooks showed that in some cases where neither country's textbooks had covered the exact topic or presented a similar problem, the Japanese students' ability to decipher the problems and attend to all their dimensions led to their success. The findings indicate that Iranian students generally succeeded in solving problems for which they had encountered a very similar example in their textbook, in addition to having the underlying content covered in their curriculum. In contrast, Japanese students demonstrated greater efficiency and reasoning ability when confronted with problems for which they had no directly similar example.

The analysis revealed that for several other problems, the significant difference in correct response rates between the two countries could be attributed to the Iranian textbooks' lack of attention to certain aspects of problem-solving and their limited range of problem scenarios. In all problems involving length measurement, unlike Japanese textbooks, Iranian textbooks do not provide students with opportunities to measure lengths where the starting point is not aligned with the zero mark on the measuring tool. Consequently, Iranian students, relying on their prior experiences, performed poorly on such problems. This finding aligns with results from other studies comparing geometry curricula and textbooks across different countries. For instance, research by Choi and Park (2013) also supports the notion that repeated exposure to identical problem types can lead to student disengagement and the misconception that the same approach applies to all situations. According to the data from the reviewed TIMSS 2019 questionnaire, which assesses students' familiarity with the seven geometry topics, it was found that 74% of Iranian students and 81% of Japanese students had been taught the geometry topics covered in TIMSS 2019 by the end of fourth-grade. It is noteworthy that this 7-percentage-point difference in the proportion of students who had studied the geometry topics was consistent across all geometry topics. Furthermore, this study only examined 8 specific problems. Therefore, this difference cannot be considered a primary reason for the performance gap in these items. To fully understand the performance differences, other factors- such as teaching methods, assessment techniques, and learning opportunities like textbooks from both countries- must be considered, as was done in the present study.

In summary, the study's findings indicate that according to the seven-category classification of the Measurement and Geometry domain in the TIMSS 2019, the average performance of Iranian students was lower than that of Japan across all topics. Furthermore, Iranian students were

generally more successful in solving problems that aligned directly with the curriculum and textbook content, and for which similar examples were available in their textbooks. In contrast, Japanese students demonstrated a greater ability to reason through and solve problems that did not have direct counterparts in their textbooks.

Based on the present study, it is essential that, in addition to ensuring the vertical coherence of topics within textbooks and their sequential development across grades, careful attention be paid to how problems are presented. For certain topics, textbooks should be designed to equip students with the skills to effectively solve problems in novel contexts and unfamiliar formats. Furthermore, the problems presented should serve to enhance students' cognitive and critical thinking levels. Various studies affirm that students must be exposed to problems of varying complexity to succeed in solving such problems in international studies (Wijaya, 2017; Choi & Park, 2013). It is also crucial that textbooks avoid providing all the necessary guidance for problem-solving in every instance, as this can deprive students of the opportunity to think independently. If multiple problems are presented for a single topic, the textbook should strive for diversity in perspective and formulation to prevent the gradual erosion of students' problem-solving abilities. This principle has been highlighted in various studies across different educational levels and subjects, and considering it, alongside other factors, can be instrumental in improving the performance of Iranian students. Notable examples of such research in the Persian literature include the works of Firouzshahi and Yaftian (2022) and Shayan (2017).

It should be noted that these differences in students' geometric performance and their success in solving problems with higher cognitive demands could stem from various other factors. These include teaching methods, classroom conditions, teacher preparation, and students' attitudes and beliefs towards mathematics (Chávez et al., 2015; Wijaya, 2017; Kiamanesh & Mohsenpour, 2010), all of which warrant further in-depth investigation in future research. A limitation of this study is that only 23 items from the TIMSS 2019 Measurement and Geometry domain had been released at the time of the research. Consequently, these 23 selected items served as the basis for comparing the Iranian and Japanese textbooks. It is recommended that after the full release of all Measurement and Geometry items from the TIMSS 2019- which typically becomes available after two cycles- future researchers conduct more comprehensive comparisons. This would provide a clearer picture of Iranian students' performance based on their textbooks and the complete set of items from this study cycle. As mentioned earlier, six different textbook series are published for primary education in Japan, with the series from Tokyo Shoseki being the most widely used in Japanese primary

schools. However, another limitation of this study was the lack of access to TIMSS data specifying whether the participating Japanese students actually used this particular textbook series.

In light of the findings and the significant role textbooks play in either alleviating or exacerbating students' difficulties in deeply understanding mathematical concepts, it is recommended that educational planners and textbook authors incorporate problems with varying cognitive levels when developing content. They should seek to include a diversity of problem designs within each topic. To this end, planners and authors could benefit from reviewing textbooks from countries that perform well in international studies and analyzing the items released from these studies.

Acknowledgement

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